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Evaluate the Rate of Contamination in Soil by Using Modified Mathematical Methods

A Thesis
Submitted to the Department of Mathematics, College of Education for pure Science / Ibn Al-Haitham, University of Baghdad as Partial Fulfillment of Requirements for the Degree of Master of Science in Mathematics

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سُورَةُ القَصَصِ

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

وَٱبْتَغِ فِيمَآ ءَاتَىٰكَ ٱللَّهُ ٱلدَّارَ ٱلْـَٔاخِرَةَ ۖ

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صَدَقَ ٱللَّهُ ٱلْعَظِيمُ

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Committee Certification

We certify that we have read this thesis entitled " **Evaluate the Rate of contamination in Soil by Using Modified Mathematical Methods** " and, as an examining Committee, we examined the student **Eiman Oday Abdulhameed** in its contents and what is connected with it, and that in our opinion it is adequate for the partial fulfillment of the requirement for the degree of master of Science in Mathematics.

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Researcher

Eiman Oday

List of Symbols

Symbol	Definition
AAS	Atomic Absorption Spectrophotometer
ADM	Adomian decomposition method
C	Solute concentration ($\frac{mg}{L}$)
Cd	Cadmium
Co	Cobalt
Cr	Chrom
Cu	Copper
C_a	Concentration of adsorbed chemical
C_x	Concentration for depth x ($\frac{\mathrm{mg}}{\mathrm{L}}$)
C_0	Initial concentration
D_L	Hydrodynamic dispersion coefficient ($\frac{\mathrm{cm}^2}{\mathrm{hr}}$)
f	Irreversible reaction decay rate
Fe	Iron
GIS	Geographic Information System
ICP- MS	Inductively Coupled Plasma-Mass Spectrometry
K_d	Distribution coefficient ($\frac{\mathrm{cm}^2}{\mathrm{Kg}}$)
Mn	Manganese
Ni	Nickel

Pb	Lead
PDE	Partial differential equation
PPM	Part per million
V	The average pore – water velocity, ($\frac{\text{cm}}{\text{hr}}$)
V_x	Darcy’s flux ($\frac{cm}{hr}$)
XRF	X-Ray Fluorescence
θ	Soil's water content
ρ_b	Soil's bulk density

Author’s Publications

[1] Tawfiq, L. N. M., Kareem A. Jasim and Abdul hmeed, E. O., (2015), Pollution of Soils by Heavy Metals in East Baghdad in Iraq, International Journal of Innovative Science, Engineering & Technology, Vol. 2, Issue 6, pp: 181- 187.

[2] Tawfiq, L. N. M., Kareem A. Jasim and Abdul hmeed, E. O., (2015), Mathematical Model for Estimation the Concentration of Heavy Metals in Soil for Any Depth and Time and its Application in Iraq, International Journal of Advanced Scientific and Technical Research, Vol. 4, Issue 5, pp:718 – 726.

[3] Tawfiq, L. N. M., Kareem A. Jasim and Abdul hmeed, E. O., (2016), Numerical Model For Estimation The Concentration of Heavy Metals in Soil and Its Application in Iraq, Global Journal of Engineering Science and Researches, Vol. 3, No. 3, PP: 75- 81.

Abstract

There are many aims in this thesis:

The first aim is to develop a model equation that describes the spread of heavy metals in soil vertically, for any depth and time then solving this model equation by using Adomian Decomposition Method (ADM).

The second aim is suggesting a numerical model which describes the spread of heavy metals in soil horizontally.

The suggested model can be used to estimates the concentration of heavy metals in soil for any time. So, this model would be a method to determine the contamination levels of some heavy metals in soil such Copper, Lead, Zinc, Cadmium, Cobalt, and Nickel with the potential for this contamination sources with their impact.

The third aim is to develop a conceptual theory of interpolation in two dimensions then illustrating the accuracy, efficiency, rapidly and easy implementation of suggested methods.

The fourth aim is to compare the performance of the suggested model with traditional methods, by estimating the concentration of heavy metals in soil, then apply them in Baghdad soils after classifying it to different zones, such as: residential, industrial, commercial, agricultural and main roads. Then, compare the results of suggested methods with the results obtained by laboratory inspecting using Atomic Absorption Spectrophotometer, X-Ray Fluorescence and Inductively Coupled Plasma-Mass Spectrometry to determine the rate of accuracy. The results of this work show that the suggested model can be successfully applied to the rapid and accuracy estimation of concentration of heavy metals in soil.

Finally, we suggested some methods for the treatment of contaminated soil by using some herbal plants.

Contents

Introduction

Nonlinear phenomena are of fundamental importance in various fields of science and engineering. The nonlinear models of real-life problems are still difficult to solve either numerically or analytically.

Recently, there has been much attention devoted to the researches for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models, especially contamination model.

A contamination can be defined generally as the presence of any material in the environment in quantities that results directly or indirectly, alone or reacting with other materials, harmful effects on health of organisms. Soil contamination can be defined as the damage and change that affects soil specifications and the nature, chemical and vital properties such that it will effect negatively in a direct or indirect way at the organisms living on its surface: humans, animals, and plants.

Many researchers studied and designed mathematical models which describe the contamination in soil for different categories and properties of soil for more details see ([35],[28], and [10]) and the references therein.

There are many researchers who studied the contamination in soil by heavy metals by using traditional methods such ([1],[2],[19], and [38]).

In recent years the researchers used parallel processing technique for determining the rate of contamination in soil such ([5], [9], [12], [14], [23], [26], [31], [39], and [43-45]).

Numerous researchers have already investigated the mobility of heavy metal in the soil amended with sewage sludge and concluded that only relatively small amount of metal were available for transport in the soil water immediately after sludge application for more details see ([16], and[37]). Different studies agreed that the main reasons of increasing the concentrations of heavy metals in soil belong to the growing of industrial activities without wide controlled studies, population growth and its consequences containing landfills places and traffic of transportations and other reasons.

This thesis studied and modified the convective–diffusive equation to represent a suitable form which describes the soil contamination by heavy metals, then using this model to estimate the concentration of heavy metals in soil. It, introduced an effective, low cost and easily implemented mathematical methods to estimate soil contamination problems. Firstly, by applying ADM as a tool to solve the modified model equation which used to estimate the concentration of heavy metals in soil for any depth and time. Also, suggesting a numerical model to estimate the concentration of heavy metals in soil which spread horizontally, then the results are compared with those obtained by the traditional laboratory using a device such as: Atomic Absorption Spectrophotometer (AAS), X-Ray Fluorescence (XRF) and Inductively Coupled Plasma-Mass Spectrometry (ICP- MS) to illustrate the accuracy and efficiency of suggested model equation.

The organization of this thesis is as follows: Chapter one, contains two sections, section one represent mathematical concepts, consisting some of the definitions, hypotheses, axioms and theorems that are needed. Section two, consists a brief overview of contamination, soil, and heavy metals that are needed throughout the thesis. Chapter two, describes and develops the mathematical model for spread of contamination through soils vertically which can be used to determine the rate of contamination by estimating the concentration of heavy metals in soil, then solve this model equation by using ADM. In chapter three, we suggested a numerical model which describes the spread of contamination through soils horizontally. Chapter four introduces the practical applications of the suggested problem to estimate the contamination in soil by heavy metals such: Copper (Cu), Lead (Pb), Cadmium (Cd), Cobalt (Co), Zinc (Zn) and Nickel (Ni) in Baghdad. Finally, chapter five contains the conclusions and future works. All algorithms in this thesis have been implemented in MATLAB version 7.12.

Chapter One

Preliminaries

1.1. Introduction

This chapter gives a brief introduction to some background ideas that are used in various places throughout the thesis. In many cases this material may be familiar; however a limited discussion is provided here in an attempt to make the thesis self-contained.

Section one consists of some mathematical concepts which will be used. In section two, a general idea about contamination of environment had been introduced, especially soil contamination by heavy metals that is needed in the formulation of the model equation.

Section One: Mathematical Concepts

This section consists some of the definitions, hypotheses, axioms and theorems which we need in this thesis.

1.1.1. Partial Differential Equations [40]

"A partial differential equation (PDE) is an equation that involving the unknown function (the dependent variable), and its partial derivatives, the dependent variable must depend on more than one independent variable".

A partial differential equation would be linear when the dependent variable's power is one even its the partial derivatives, and the coefficients of the dependent variable and of each partial derivative is constant or independent variables. Otherwise, the equation is known as nonlinear.

The solution of a PDE appears as a function that satisfies the equation which discuss and satisfies the given conditions. These conditions are in two types:

- **Initial conditions** when the initial values of the dependent variable should be given at the independent variable.
- **Boundary conditions** when the dependent variable had been prescribed at the independent variable at the boundary of the domain. In this case, the boundary data is called *boundary conditions*.

There are three types of boundary conditions which are given and defined as follows:

1. **Dirichlet Boundary Conditions**: when the function is known on the boundary for a bounded domain.
2. **Neumann Boundary Conditions**: A condition that depends on the values of derivative of the function at the boundary of the domain.
3. **Robin Boundary Conditions**: A linear combination of the dependent variable and its derivative is known on the boundary [24].

Note, "a PDE with initial condition given in the one independent variable and boundary condition given in the others is said to be PDE with *initial-boundary condition* or *initial-boundary* PDE"[40].

1.1.2. "Existence and Uniqueness [46]

The existence and uniqueness theory plays an important role in analyzing approximate methods for solving PDE′s, therefore, the main theorems are introduced here while its proof is given in".

"**Theorem 1.1 (Existence)** [46]

Let U, V be Hilbert spaces and L: U → V be a bounded linear operator. Then R(L) = V if and only if R(L) is closed and if $R(L)^{\perp} = \{0\}$".

"**Theorem 1.2 (Existence and uniqueness)** [46]

Let U, V be Hilbert spaces and L: U → V be a closed linear operator. Suppose that there exists a constant C > 0 such that:

$\|Lv\|_V \geq C\|v\|_U$, for all v ϵ U (coercivity estimate).

If $R(L)^{\perp} = \{0\}$, then the operator equation Lu = f has a unique solution.

<u>Note,</u> the theorem guarantees that the general PDE will always have (existence of) exactly one (uniqueness) solution".

1.1.3. "Stability

The solution to a differential equation is said to be stable if small changes in the initial conditions, boundary conditions, or coefficients in the equation itself lead to small changes in the solution.

There are many different types of stability that are useful (for more details see " [19], and [27]).

1.1.4. " Well-Posed

A partial differential equation is called well-posed in the sense of Hadamard (1865-1963), if

- The solution exists.
- The solution is unique.

- The solution is stable, that is, the solution depends continuously on the existing data (initial and boundary conditions) "[27].

A problem is called ill posed when it is not well posed. In this case, there may not be a solution, there might be more than one solution, or (by an approximate scheme) that may not reach to the actual solution.

There are many methods to solve PDE, some of these give exact solutions others give approximate solutions: numerical or analytic, in this thesis the PDEs are handled by the ADM. Since this method is proved to be effective, powerful, and can easily implemented ([15], [16], and [26]).

1.1.5. Adomian Decomposition Method

Adomian Decomposition Method (ADM) is one of the decomposition methods provide the solution in an infinite series form; where the exact solution exists and the obtained series may converge to a closed form solution. If the exact solution does not exist, then the truncated series may be used for numerical purposes. "ADM is a creative and effective method for exactly solving functional equations of various kinds. It is important to know that a large amount of research work has been devoted to the application of ADM to a wide class of linear or nonlinear, ordinary or partial differential equations"[20]**.**The idea of this method is demonstrating the unknown function *u(x, y)* of any equation in expanded series as a sum of infinite number of terms defined by the decomposition series which can be written as:

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y), \tag{1.1}$$

where $u_n(x, y)$, $n \geq 0$ are to be determined in a recursive manner.

Now, we summarize ADM to solve the linear differential equation by the following steps:

1. Writing the PDE in operator form by:

$$Lu + Ru = g, \tag{1.2}$$

where L is the lower order derivative which is assumed to be invertible, R is other linear differential operator, and g is a source term.

2. Applying the inverse operator L^{-1} to both sides of equation (1.2) then using the given condition to get:

$$u = f - L^{-1}(Ru), \tag{1.3}$$

where f is the function represents the terms arising from integrating g and by using the given conditions that are assumed to be prescribed.

3. The components u_0, u_1, u_2, … , are recurrently determined. Then substituting (1.1) into both sides of (1.3) to get:

$$\sum_{n=0}^{\infty} u_n = f - L^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n\right)\right) \tag{1.4}$$

Rewrite equation (1.4) as follows:

$$u_0 + u_1 + u_2 + u_3 + \cdot\cdot\cdot = f - L^{-1}\left(R(u_0 + u_1 + u_2 + \cdot\cdot\cdot)\right) \tag{1.5}$$

4. Determining the recursive relation as follows:

$u_0 = f$,

$$u_{k+1} = -L^{-1}\left(R(u_k)\right),\ k \geq 0, \tag{1.6}$$

or equivalently:

$u_0 = f$,

$u_1 = -L^{-1}(R(u_0)),$

$u_2 = -L^{-1}(R(u_1)),$

$$u_3 = -L^{-1}(R(u_2)), \tag{1.7}$$

5. Then substitute (1.7) into (1.1) to get the solution in a series form.

1.1.6. Interpolation

Interpolation is the approximation which can be used to determine unknown value of a point by using known value of neighboring points where the point belong in the bounded, continuous and smooth domain. there is high correlation between neighboring data points or phenomenon.

Interpolation can be used for estimating the values on a continuous grid based model.

In this section, we shall consider the interpolator approximation, in which Weierstrass approximation theorem guarantee that one can always find a polynomial that is arbitrarily close to a given function on some finite interval. This means that the approximation error is bounded and can be reduced by the choice of the adequate polynomial. Unfortunately this theorem is not a constructive one, i.e., it does not present a way how to obtain such a polynomial, i.e., the interpolation problem can also be formulated in another way viz. as the answer to the following question: How to find a good representative of a function that is not known explicitly, but only at some points of the interested domain.

"Theorem 1.3 (Weierstrass Approximation Theorem) [11]

Let f(x) be a continuous function on [a, b], then for any $\varepsilon > 0$, there exist an integer n and polynomial $P_n(x)$ of degree n, such that:

$$| f(x) - P_n(x) | < \varepsilon, \quad \forall x \in [a, b] \text{ "} \tag{1.8}$$

Section Two: Contamination

This section consists of a brief overview of environmental contamination, soil, heavy metals which we need throughout this thesis.

1.2.1. Environment Contamination

A contamination can be defined generally as "the presence of any material in the environment in quantities that results directly or indirectly, alone or reacting with other materials, harmful effects on health of organisms" [36].

Environmental impact on human health is the most obvious aspect that is considered in every study, including the quality of life and what changes in the environment and dangerous on human life, using biological, physical, chemical, and psychosocial factors to show damage how far it can go.

The three main parts that researchers partition the environment are water, air, and soil, each one of them represent a wide field that needs extended studies to be covered as it deserves. At the end the three main parts are not separated and any type of pollution in any one of these three parts will affect the other two parts.

1.2.2. Soil Contamination

The contamination in soil is defined as the presence of these metals how they build up and their travelling quickness in the soil which depends on many factors. Some contaminants undergo chemical changes specially those organic (carbon- based), they also may degrade in the soil which may change their toxicity negatively or positively, compared with the original compound. Some chemical elements (such as metals) may have some changes in their characteristics, although they cannot break down, but these changes would help to make them uptake by plants and animals easily.

The contaminants are varying in their ability to:

♦ End up in water that held in the soil or in the underlies groundwater leaching through the soil;

♦ Evaporating into the air; or

♦ Bind tightly to the soil compounds.

The soil characteristics also affect the contaminants fates, whether they can be readily taken up by plants or animals. Site managing and using land in various practices can affect some soil characteristics. "Important soil characteristics that may affect the behavior of contaminants include:

♦ Soil mineralogy and clay content (soil texture);

♦ PH (acidity) of the soil;

♦ Amount of organic matter in the soil;

- ♦ Moisture levels;
- ♦ Temperature; and
- ♦ Presence of other chemicals" [18].

In this thesis, we focus on soil contamination by heavy metals.

1.2.3. What are the Heavy Metals?

"Means all heavy metals that increase density 5 g/cm^3, and at density less it is called light metals"[42]. Some of these metals are having different biological effects, they play an important role in the lives of the living. For example the iron well-known enzymes in the blood even all of the elements manganese how their installation is important, zinc, and copper enzymatic catalysts. However, those metals are toxic and dangerous to be in specified concentrations. Adding to that the seriousness of these metals, it is not possible to be analyzed by bacteria and other natural processes as well as the authenticity of which enable them to spread over long distances for ton or sources sites. Perhaps the most dangerous thing is due to susceptibility to each bio-accumulate in the tissues and organs of living organisms in the environment water or land. In addition, some heavy metals serve as radioactive isotopes, therefore, "these metals will be charged double the risk to the environment in terms of being toxic and radioactive at the same time, as is the case in 65 of radioactive zinc, uranium 235"[25].

"Heavy metal contamination in soil may pose risks and hazards to human beings. Excessive concentrations of some heavy metals in biological systems, especially animals (human beings in particular) are highly dangerous to human health, and may even cause death. For example, heavy metals such as: Cadmium (Cd), Nickel (Ni) and Arsenic (As) are carcinogenic. Figure (1.1) illustrates the distribution of heavy metals compounds in soil as given in" [13]. "Table (1.1) gives a summary of some dangerous heavy metals that are commonly present in farm soils and their health impacts to human beings" [34].

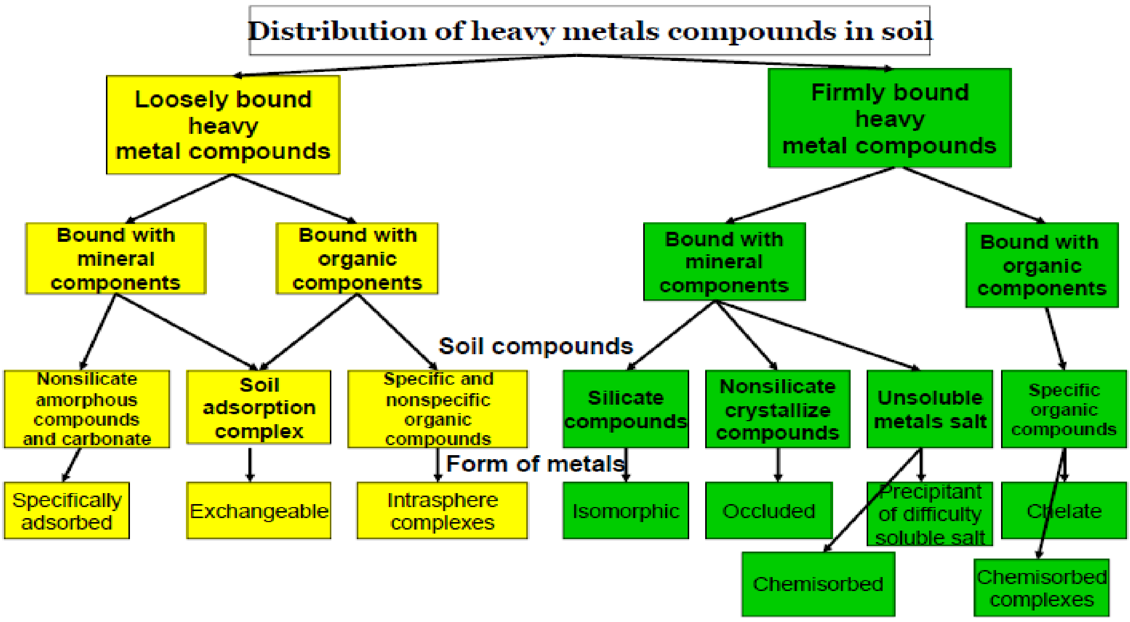


Figure 1.1: Distribution of heavy metals compounds in soil.

Table 1.1: Some dangerous heavy metals and their health impacts to human beings[7]

Heavy Metal	Health Impact/s
Pb	Mental lapse or even death
Cr	Allergic dermatitis
As	Skin damage, cancer, affects kidney and central nervous system
Zn	Zinc shortages can cause birth defects
Cd	Affects kidney, liver and GI tract
Cu	Anemia, liver and kidney damage, and stomach/intestinal irritation
Ni	Various kinds of cancer

The sources of some heavy metals in soils as follows:[36]

"Cadmium: metal plating - Paints - Sticky plastic - the battery industry.

Copper: coal mining and its aftermath - fertilizer.

Lead: fuel and coal combustion - iron and steel production plants.

Nickel: oil and coal - alloying - coating of metal combustion.

Zinc: iron and galvanized steel - alloys - Batteries – rubber factories".

Chapter Two

Formulation of Mathematical Model to Evaluate the Contamination in Soil for Any Depth

2.1. Introduction

In this chapter, a mathematical model for spread of contamination through soils had been described and developed, which can be used then to determine the rate of contamination by estimating the concentration of heavy metals in soil. We use ADM to solve this model equation where the solution of the equation represents the concentration of heavy metals in soil for any depth and time without spending in traditional laboratory inspecting.

2.2. Mathematical Model

Mathematical model is a simplified representations of some real world entity. It's a simplified mathematical construct related to a part of reality and created for a particular purpose.

"Also it can be defined as that model which based on the mathematical relations and describe the behavior of a phenomenon.

To build a mathematical model for a problem, it needs to study and understand the problem, then design the model for more details see" [32].

2.3. Convective – Diffusive Equation

For many years, potentially harmful substances have been added to the soil through land application of agricultural chemicals, industrial wastewater and sludge disposal, landfills, and leaking hazardous waste storage sites. The potentially harmful substances including heavy metals, pesticides and other industrial organic chemicals, and even plant nutrient supplements may contaminate soils, surface water bodies, and subsurface aquifers, Amacher et al. 1986 [4]. This, interest about the quality of both soil and water had promoted to more concerns on studying and describing the processes of solute reactions and mobility in soils.

In most of the studies in this field, models including soil texture reactions including retention and release are needed. Operations like retention and release in soil are diversified such as exchanging ion, adsorption/desorption and dissolution/precipitation. "Retention and release are influenced by number of soil properties including texture, bulk density, power of hydrogen (PH), Electric Conductivity (EC), organic matter, and type and amount of clay minerals. Then the model equation which describes this problem is said to be convective – diffusive equation and has the form" [35]:

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(D_L \frac{\partial C}{\partial x}\right) - f - \frac{\partial C_a}{\partial t} \qquad (2.1)$$

Where;

C: Solute concentration ($\frac{mg}{L}$)

v: Darcy's flux ($\frac{cm}{hr}$)

D_L: Hydrodynamic dispersion coefficient ($\frac{cm^2}{hr}$)

x: Soil depth (cm)

t: Time (day^{-1})

C_a: Concentration of adsorbed chemical

f: Irreversible reaction decay rate.

2.4. Modification of Mathematical Model

The convective – diffusive equation described in the previous section consists of description of the contamination of soils by sewage sludge. Heavy metals released from sewage sludge are distributed throughout the soil system remain in the soil solution as iron, organic and inorganic complexes. Some of these heavy metals are mobile for uptake by plants. This mobility and availability depends on several factors including soil texture and PH (Nouri., 1980 [29] and Alloway., 1998 [3]). Several researchers have already investigated the mobility of heavy metals in the soil amended with sewage sludge and concluded that only relatively a small amount of metal was available for transport in the soil's water immediately after sludge application which added to the model equation (2.1) as a modification. Also, in the industrial regions, where some types of factories are active, several chemical and petrochemical processes would be also active. As a result, industrial water becomes contaminated with various substances which are harmful, these are sources of environmental contamination, which added in the description of the modified model equation. Then, the model equation (2.1), can be written as follows: [29]

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x} - \frac{\rho_b}{\theta} \frac{\partial S}{\partial t} \qquad (2.2)$$

Where;

$S = k_d . C$

C: Solute concentration ($\frac{mg}{L}$)

K_d : Distribution coefficient ($\frac{cm^2}{Kg}$)

θ: Soil's water content

ρ_b: Soil's bulk density

V_x: Darcy's flux ($\frac{cm}{hr}$)

D_L: Hydrodynamic dispersion coefficient ($\frac{cm^2}{hr}$)

x: Soil depth (cm)

t: Time (day^{-1})

Simply, equation (2.2) can be written as follows:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x} - \frac{\rho_b}{\theta} \frac{\partial}{\partial t}(K_d C) \qquad (2.3)$$

Since K_d is a constant, so rewrite equation (2.3) as follows:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x} - \frac{K_d \rho_b}{\theta} \frac{\partial C}{\partial t} \qquad (2.4)$$

and after rearranging we get:

$$\frac{\partial C}{\partial t} + \frac{K_d \rho_b}{\theta} \frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x} \qquad (2.5)$$

That is:

$$(1 + \frac{K_d \rho_b}{\theta}) \frac{\partial C}{\partial t} = D_L \frac{\partial^2}{\partial x^2} C - V_x \frac{\partial}{\partial x} C \qquad (2.6)$$

Let

$$\rho = \left(1 + \frac{K_d \rho_b}{\theta}\right) \qquad (2.7)$$

So, equation (2.6) can be written as:

$$\rho \frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x}, \quad 0 < x < \infty, \quad t > 0 \qquad (2.8)$$

and $\rho \approx 1$, then equation (2.8) can thus be written as (for simplicity $D_L = D$, and $V_x = V$):

$$\frac{\partial C}{\partial t} = D\ \frac{\partial^2 C}{\partial^2 x} - \ V\ \frac{\partial C}{\partial x} \qquad (2.9a)$$

which is a second order linear PDE, with initial - boundary conditions.

Now, how to choose initial and boundary conditions. A variety of conditions may be specified depending on the type of soil. There are different types of soil depending on properties, structure, characteristic or layers. In this thesis we classify the soil depending on characteristic and properties of soil as follows: Sand (fine and coarse), Silt, Loam, Crags and Clay [17] with physicochemical properties of the soil given in chapter four (Table 4.9).

If the soil is a loam land then the conditions are:

$$C(x, 0) = C_{x,0} = C_0\ e^{\frac{-Vx}{D}}$$

$$C(0, t) = C_0 \ \text{ and } \ \frac{\partial C}{\partial x}(\infty, t) = 0, \qquad (2.9b)$$

Where;

C_0: Initial concentration.

C_x: Concentration for depth x ($\frac{mg}{L}$)

V: The average pore – water velocity, ($\frac{cm}{hr}$)

x: Soil depth (distance) (cm).

t : Time (day^{-1}).

The amount of each element retained by each soil ($\frac{\text{mg}}{\text{kg}}$) was calculated from the initial concentration in solution ($\frac{\text{mg}}{\text{L}}$) and the final concentration C in solution ($\frac{\text{mg}}{\text{L}}$). Equation (2.9), which can be represented as a mathematical model for spread of contamination through soil which can be used to determine the rate of contamination. The solution of model equation gives the concentration of the heavy metals in soil for any depth and time. This model can be considered as an important model to give concentrations of heavy metals without spending in traditional laboratory inspecting.

2.5. Solution of Model Equation

We will use the ADM which have been discussed in chapter one to solve the equation (2.9).

First we consider the linear differential equation in an operator form by

$$L_t\, C = D\, L_{xx}\, C - V\, L_x\, C \qquad (2.10)$$

where L, is the differential operator which is invertible. So, applying the inverse operator L^{-1} with respect to t, both sides of equation (2.10), and using the initial condition: $C(x, 0) = C_{x,0} = C_0\, e^{\frac{-Vx}{D}}$

We get:

$$C = C_0\, e^{\frac{-Vx}{D}} + D\, L_t^{-1}\, L_{xx}\, C - V\, L_t^{-1}\, L_x\, C \qquad (2.11)$$

Depending on ADM, the solution C can be defined by an infinite series of components given by:

$$C = \sum_{n=0}^{\infty} C_{x,n}$$

Then the equation (2.11) can be written as:

$$\left(\sum_{n=0}^{\infty} C_{x,n}\right) = C_0\, e^{\frac{-Vx}{D}} + D\, L_t^{-1}\, L_{xx} \left(\sum_{n=0}^{\infty} C_{x,n}\right) - V\, L_t^{-1}\, L_x \left(\sum_{n=0}^{\infty} C_{x,n}\right) \qquad (2.12)$$

where $C_{x,0}$, $C_{x,1}$, $C_{x,2}$, …, can be determined as recurrent. As given in our model, the zeroth component $C_{x,0} = c_0\, e^{\frac{-Vx}{D}}$, then:

$$C_{x,1} = L_t^{-1}\left(D\, L_{xx}\, c_0\, e^{\frac{-Vx}{D}} - V\, L_x\, c_0\, e^{\frac{-Vx}{D}}\right)$$

$$= L_t^{-1}\left(c_0\, \frac{V^2}{D}\, e^{\frac{-Vx}{D}} + c_0\, \frac{V^2}{D}\, e^{\frac{-Vx}{D}}\right)$$

$$= 2t\, c_0\, \frac{V^2}{D}\, e^{\frac{-Vx}{D}}$$

$$C_{x,2} = L_t^{-1}\left(D\, L_{xx}\left(2t\, c_0\, \frac{V^2}{D}\, e^{\frac{-Vx}{D}}\right) - V\, L_x\left(2t\, c_0\, \frac{V^2}{D}\, e^{\frac{-Vx}{D}}\right)\right)$$

$$= L_t^{-1}\left(2t\, c_0\, \frac{V^4}{D^2}\, e^{\frac{-Vx}{D}} + 2t\, c_0\, \frac{V^4}{D^2}\, e^{\frac{-Vx}{D}}\right)$$

$$= \frac{4\, t^2}{2}\, c_0\, \frac{V^4}{D^2}\, e^{\frac{-Vx}{D}}$$

$$C_{x,3} = L_t^{-1}\left(D\, L_{xx}\left(\frac{4\, t^2}{2}\, c_0\, \frac{V^4}{D^2}\, e^{\frac{-Vx}{D}}\right) - V\, L_x\left(\frac{4\, t^2}{2}\, c_0\, \frac{V^4}{D^2}\, e^{\frac{-Vx}{D}}\right)\right)$$

$$= L_t^{-1}\left(\frac{4}{2}t^2 c_0\ \frac{V^6}{D^3}\ e^{\frac{-Vx}{D}} + \frac{4}{2}t^2 c_0\ \frac{V^6}{D^3}\ e^{\frac{-Vx}{D}}\right)$$

$$= 8\,\frac{t^3}{3!}\ c_0\ \frac{V^6}{D^3}\ e^{\frac{-Vx}{D}}$$

And so on for other components. Consequently, the solution in a series form is given by:

$$C = C_{x,0} + C_{x,1} + C_{x,2} + C_{x,3} + \ldots\ldots$$

$$= c_0\ e^{\frac{-Vx}{D}} + 2t\ c_0\ \frac{V^2}{D}\ e^{\frac{-Vx}{D}} + \frac{4\ t^2}{2}\ c_0\ \frac{V^4}{D^2}\ e^{\frac{-Vx}{D}} + 8\frac{t^3}{3!}\ c_0\ \frac{V^6}{D^3}\ e^{\frac{-Vx}{D}} + \ldots.$$

That is:

$$C = c_0\ e^{\frac{-Vx}{D}}\{\ 1+\left(\frac{t}{D}\right)(2v^2) + \frac{1}{2!}\left(\frac{t}{D}\right)^2(2v^2)^2 + \frac{1}{3!}\left(\frac{t}{D}\right)^3(2v^2)^3 + \ldots\}$$

i.e., $C = c_0\ e^{\frac{-Vx}{D}}\ e^{\frac{2tV^2}{D}}$

Thus

$$C = c_0\ \exp\left(\frac{-Vx}{D} + \frac{2V^2}{D}t\right) = c_0\ \exp\left\{\frac{V}{D}(-x + 2Vt)\right\} \qquad (2.13)$$

Thus, equation (2.13) represents the closed form solution of the model equation.

Now, we choose D = 0.5 $\frac{m^2}{d}$ and V = 5.14 $\times 10^{-6}$ ms^{-1} $= 44.4096 \times 10^{-2}\ \frac{m}{d}$, depends on results of ([15], and [39]).

Therefore, $C = c_0 \exp\{\frac{44.4096\times10^{-2}}{0.5}\ (-x + 2(44.4096 \times 10^{-2})t)\}$

$$C = c_0 \exp\{0.888192(-x + 0.888192t)\}, \qquad (2.14)$$

Equation (2.14) represents the concentration of heavy metals in soil for any depth x and time t.

2.6. Results and Discussion

For testing the efficiency of suggested model equation (2.14) we applied on Baghdad soil and Figure (2.1), illustrate the results of the concentrations of heavy metals in soil of Baghdad city for times in days, which had been calculated from equation (2.14). Figure (2.2), illustrates the comparison for concentration of heavy metals in Baghdad soil with previous studies, we see that:

- **Lead (Pb)**: We can deduce that the soil of some areas of Baghdad under the influence of lead contaminated absolutely and industrial plants such as the smelter and battery plant in Abu Ghraib and Waziriya and brick factories polluting to the air and soil existing in the Adhammaai region, as well as the impact of vehicles with gasoline, which took increasing dramatically, leading to increased air pollution in lead and thus contamination of soil, as well as the impact of foundries and workshops with public and private sector in the city areas.

- **Zinc (Zn):** The reason for increased concentrations of zinc in the soil of Baghdad city to increase the acidic soil because of the easy assimilation of zinc, as well as a result of increased soil organic matter,

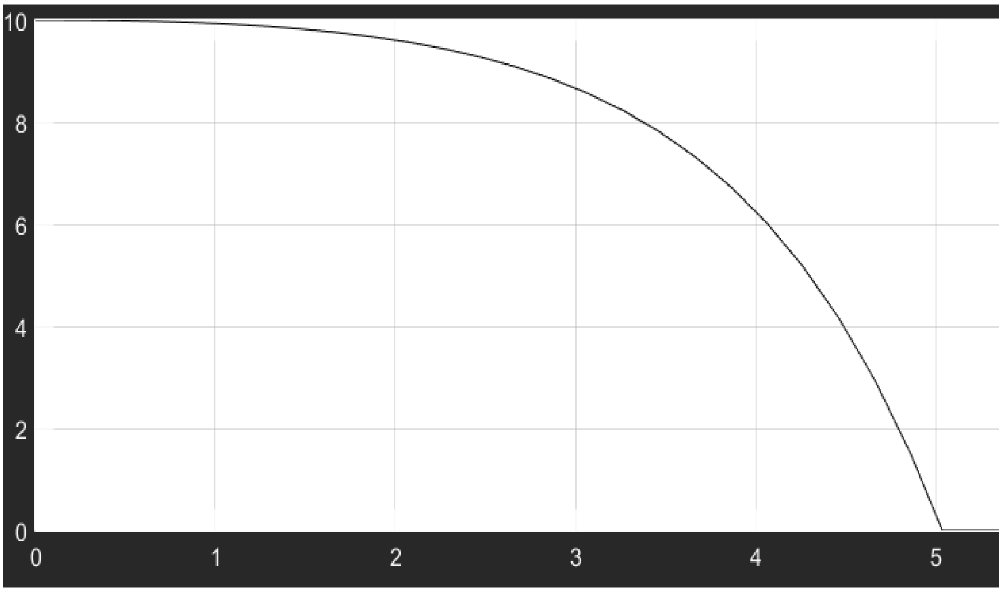
  and as indicated in several studies have suggested that increasing concentrations of zinc in the soil due to the influence of vegetation as well as the impact of human activities and of laboratories and foundries and the use of Nutraceuticals and pesticides in the soil.

- **Copper (Cu):** We see that the soil of some area is contaminated with copper significantly and there is also this element pollution in other areas such as Waziriya and Rashid Hospital. The cause of increasing in the concentrations of copper in the soil of the city back to adsorption by clay minerals and its transmission over long distances with the river sediment as well as increasing organic materials, and increased copper in the soil due to industrial activities such as workshops, foundries and smelting operations in addition to the impact of irrigation water and drainage.

- **Nickel (Ni):** We see that the soil of some areas had been contaminated with nickel. The cause of the increased concentrations of nickel in the soil due to adsorption processes by clay minerals and to the basic compounds presence in the rocks and sedimentary rocks, and organic materials play an important role in increasing concentrations of nickel in the soil. Industrial activities also play an equally important role in increasing concentrations of nickel in the soil through the spread of the electrical industry and laboratories as well as batteries, workshops and foundries with the private sector and deployed in the city.

- **Cobalt (Co):** We conclude that some of the city contaminated soils crusts. The increase in the concentrations of cobalt in the soil is the result of several factors, including the origin and composition of soil and weathering processes and the impact of human activities as well as the impact of irrigation water.
- **Cadmium (Cd):** We see that the soil's city contaminated with cadmium. Increasing concentrations of cadmium in the soil due to the increase of organic matter in the soil and the presence in the rock base and as a result of industrial activities as well as the use of conditioners, pesticides, and the impact of water drainage.

C(mol/L)



Time (Day)

Figure 2.1: Concentrations of heavy metals for time t(d) and depth x(cm) in Baghdad city.

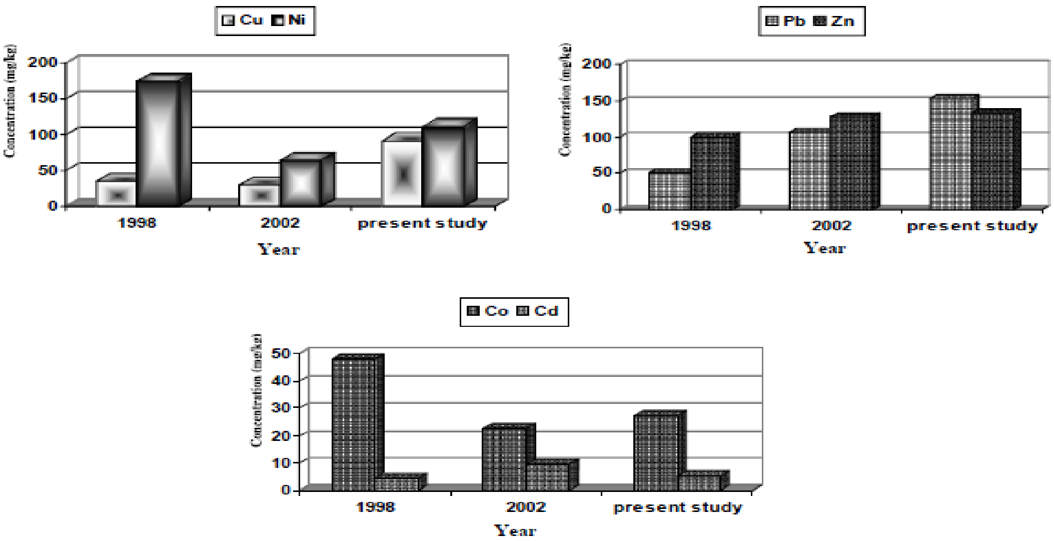


Figure 2.2: Comparison for concentration of heavy metals in Baghdad soil with previous studies

Chapter Three

Formulation of Numerical Model to Evaluate the Contamination in Soil for Any Neighborhood

3.1. Introduction

In this chapter, an interpolation technique will be used to determine the rate of contamination in the soil surface by some heavy metals, based on determining the concentration of each studied metal and then comparing it with the global scale. The bilinear interpolation method will be implemented since it has the property that give high order of accuracy with the same given data in the domain and, it agrees with the exact function. Also, we used two-dimension interpolation with its modification.

3.2. Bilinear Interpolation

Let function *f*(*x*,*t*) be defined at the point P_1, P_2, P_3 and P_4 which represent the vertices of a rectangular region. Let P be any point inside this region. Through *P* draw horizontal and vertical lines intercepting the sides of the region at points *A*, *B*, *C* and *D*. Define the numbers α, β, $\gamma$and δ as (illustrated in Figure (3.1)):

$$\alpha = \frac{AP}{AB}, \beta = \frac{BP}{AB}, \gamma = \frac{CP}{CD}, \delta = \frac{DP}{CD} \qquad (3.1)$$

Applying interpolation rule for sides P_1P_2 and P_3P_4 then for line segment *CD*, we can obtain the following ***bilinear interpolation*** formula [13]:

$$L(P) = \delta\beta\, f(P_1) + \alpha\delta\, f(P_2) + \beta\gamma\, f(P_3) + \alpha\gamma\, f(P_4) \qquad (3.2)$$

which uses resampling two linear interpolations, by using the some nearest coordinates or two variable vertices to estimate another value which lie among them. It can be expressed as an extension of linear interpolation for interpolating functions of two variables on a regular two dimensions grid, with a key idea of performing linear interpolation in one direction, then again in the other direction. The result is an interpolation which is not linear, although each step is linear. It should be noted that it doesn't make any difference whether we start interpolating with the values of any of the two dimensions followed by the values of the other one; the solution will be the same.

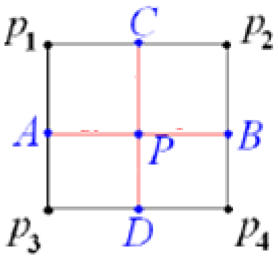


Figure 3.1: Bilinear Interpolation

To understand the topic more clearly, assume that we have the value of the concentration for a specific metal at four given positions $P_{11}=(x_1,t_1)$, $P_{12}=(x_1,t_2)$, $P_{21}=(x_2,t_1)$, and $P_{22}=(x_2,t_2)$, where x represent the distance and t represent the time, then the interpolate function from these positions $C=f(x,t)$ represent the concentration of heavy metals for any position in this region which illustrated in Figure (3.2), the idea is using linear interpolation in one direction, and repeat using linear interpolation in the other direction.

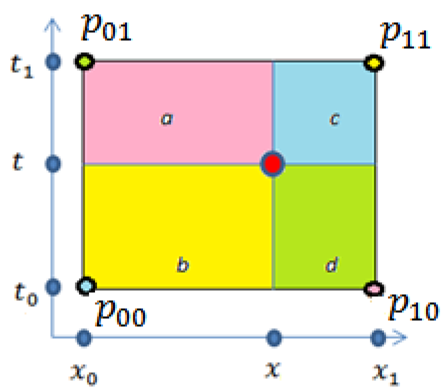


Figure 3.2: The Region of Bilinear Interpolation

Geometrically, as in Figure (3.3), we need to find the value of the function f(x, t) at the red spot which equals to the sum of the product of each coloured spot by the area of the same colour rectangle, divided by the area of all four rectangles.

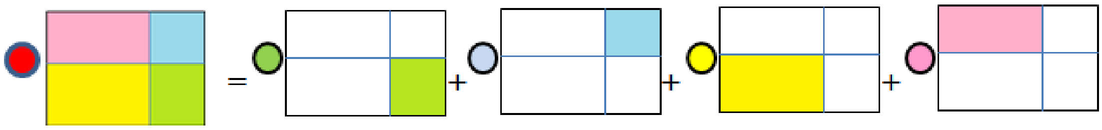


Figure 3.3: Geometrically interpretation of Bilinear Interpolation

Then the interpolation formula can be written as:

$$f(x,t) = \sum_{i=0}^{1}\sum_{j=0}^{1} P_{ij}x^i t^j = P_{00} + P_{10}x + P_{01}t + P_{11}xt \qquad (3.3)$$

Where P_{ij}, i, j= 0, 1; is the concentration of the heavy metal under studying at the point (x_i, t_j).

As stated above, the bilinear equation represents an equation of a surface. In other words, the left hand side of the equation is information, which could represent elevation value, pixel's brightness, temperature, or texture map. This information is expressed as a function of the (x, t) to find the parameters of the bilinear transformation. This bilinear equation does include information to constrain an (x, t) solution. The (x, t) are known from gridding or a regular grid, which is identical to our work, because we have a function of two variables (distance *x* and time *t*) represents the equation of surface, and we want to find the value of concentration of specific metal at the soil surface.

To interpolate the P_{00} value, it is clearer to be done by putting the points P_{11}, P_{12}, P_{21}, and P_{22} in a Table (3.1). These four points' coordinates are needed to be entered, x_0 and t_0 define the points to be performed from the interpolation, and then P_{00} is the interpolated value which represents the solution.

Table 3.1: how to choose interpolate data

	x_1	x_0	x_2
t_2	p_{12}		p_{22}
t_0		p_{00}	
t_1	p_{11}		p_{21}

3.3. Two Dimensions Lagrange Interpolation Technique

Let $(x_0, t_0), (x_1, t_1), \ldots, (x_m, t_n)$ be the given points in the (x, t) plane, we have to find a polynomial passing through these points, where (x_i, t_j), $i = 0,1,2,\ldots..m$, and t_j, $j=0,1,2\ldots,n$, are given as a rectangular grid.

The polynomial of degree m in x, and n in t, can be constructed to interpolate through (x_i, t_j) as follows:

Let $\ell_i(x) = \prod_{\substack{k=0 \\ k\neq i}}^{m} \frac{(x-x_k)}{(x_i-x_k)}, \qquad \ell_j(t) = \prod_{\substack{j=0 \\ k\neq j}}^{n} \frac{(t-t_k)}{(t_j-t_k)}$

Let $\ell_{ij} = \ell_i(x)\ell_j(t)$, $0 \leq i \leq m$, $0 \leq j \leq n;$

So, we have $\ell_{ij}(x_k, t_s) = \begin{cases} 1 & i = k, j = s \\ 0 & otherwise \end{cases}$

Thus ℓ_{ij} represents a polynomial of degree m in x and n in t. Then, we get the two dimensions–Lagrange interpolating polynomial $\mathcal{P}_{m,n}(x, t)$ that interpolates f(x, t) in the given data would be written as follows:

$$\mathcal{P}_{m,n}(x, t) = \sum_{i=0}^{m} \sum_{j=0}^{n} f(x_i, t_j)\ell_{ij}(x, t),$$

$$= \sum_{i=0}^{m} \sum_{j=0}^{n} f(x_i, t_j) \prod_{\substack{k=0 \\ k \neq i}}^{m} \frac{(x - x_k)}{(x_i - x_k)} \prod_{\substack{j=0 \\ k \neq j}}^{n} \frac{(t - t_k)}{(t_j - t_k)} \qquad (3.4)$$

Now, we introduce theorems which guaranty the uniqueness of these polynomials and their proofs are given in [11].

Theorem 3.1

There exist a unique polynomial $\mathcal{P}_{m,n}(x, t)$ passing through given $(n+1)(m+1)$ data.

Note: The problem is well-posed; *i.e.*, it has a unique solution that depends continuously on the data.

Theorem 3.2

Suppose (x_i, t_j), $0 \leq i \leq m$, $0 \leq j \leq n$, are distinct points in the interval [a, b]×[c, d] and $f \in C^{m+n+2}$ ([a, b]×[c, d]). Then

$$R(x,t) = f(x,t) - \mathrm{P}_{m,n}(x,t)$$

$$= \frac{\partial^{m+1} f(\theta_1, t)}{(m+1)!\,\partial x^{m+1}} + \frac{\partial^{n+1} f(x, \theta_2)}{(n+1)!\,\partial t^{n+1}} - \frac{\partial^{m+n+2} f(\xi_1, \xi_2)}{(m+1)!\,(n+1)!\,\partial x^{m+1}\partial t^{n+1}}$$

It is clear that finding the error is practically difficult with the two dimensions interpolation. But we can determine the behavior of the errors.

Notification: Many authors specialize the formula (3.4) for small numbers of data, before asserting that certain shortcomings make it a bad choice for practical computations. Among the shortcomings sometimes claimed are these:

1. Each evaluation of $\mathrm{P}(x,t)$ requires $\mathcal{O}(n^2)$ and $\mathcal{O}(m^2)$.
2. Adding a new pair $((x_{m+1}, t_{n+1}), f_{m+1,n+1})$ requires a new computation from scratch.
3. The computation is numerically unstable.

For all these reasonable statements, a suggested modification were introduced at the following paragraph.

3.4. Suggested Modification

To avoid the previous limitation we suggest the following modification.

Firstly in equation (3.4) let: $w_m(x) = (x - x_0) \ldots (x - x_m)$;

rewrite $w_m(x)$ as follows:

$$w_m(x) = (x - x_i)\ \{(x - x_0)\ \dots\ (x - x_{i-1})(x - x_{i+1})\dots(x - x_m)\}$$

so, $$w'_m(x_i) = \prod_{\substack{k=0 \\ k\neq i}}^{m} (x_i - x_k) \tag{3.5}$$

then, $$\ell_i(x) = \prod_{\substack{k=0 \\ k\neq i}}^{m} \frac{(x-x_k)}{(x_i-x_k)} = \frac{w_m(x)}{(x-x_i)w'_m(x_i)}$$

Also, let $v_n(t) = (t - t_0)(t - t_1)\dots(t - t_n)$

$$= (t - t_j)\{(t - t_0)\dots(t - t_{j-1})(t - t_{j+1})\dots(t - t_n)\}$$

so, $$v'_n(t_j) = \prod_{\substack{k=0 \\ k\neq j}}^{n} (t_j - t_k) \tag{3.6}$$

then, $$\ell_j(t) = \prod_{\substack{k=0 \\ k\neq j}}^{n} \frac{(t-t_k)}{(t_j-t_k)} = \frac{v_n(t)}{(t-t_j)v'_n(t_j)}$$

So, the polynomial will be written as:

$$Ƥ_{m,n}(x, t) = \sum_{i=0}^{m}\sum_{j=0}^{n} f(x_i, t_j)\frac{w_m(x)}{(x-x_i)w'_m(x_i)} \bullet \frac{v_n(t)}{(t-t_j)v'_n(t_j)}$$

That is,

$$Ƥ_{m,n}(x,t) = w_m(x)v_n(t)\ \sum_{i=0}^{m}\sum_{j=0}^{n} f(x_i, t_j)\frac{1}{(x-x_i)w'_m(x_i)} \bullet \frac{1}{(t-t_j)v'_n(t_j)} \tag{3.7}$$

What about updating?

Looking at (3.7), the calculation will be reduced by taking common factor. So this modification can consider a powerfully technique when the given data is specific. Others we need vast memory since we have two calculations:

1. Divide each $w_m(x)$, by $(x - x_i)$ and $v_n(t)$ by $(t - t_j)$; i=0,…,m, j=0,…,n. (one flop for each point) for a cost of (m+1) flops and (n+1) flops.

2. Compute $w'_m(x_i)$ and $v'_n(t_j)$ with formula (3.5) and (3.6) for the each (*n*+1) and (*m*+1) flops.

The formula (3.7) can also be developed with (*n*) and (*m*) flops by applying this development recursively while minimizing the number of divisions, we get the following algorithm for computing $w_m(x)$, $w_n(t)$:

$w_0^{(0)} = 1$

for *i*=1 to *m* do

for k=0 to *i*-1 do

$w_k^{(i)} = (x_k - x_i)w_k^{(i-1)}$

end

$w_i^{(i)} = \prod_{k=0}^{i-1}(x_i - x_k)$

end

for *i*=0 to *m* do

$w_i^{(i)} = w_k^{(i)}/(x - x_i)$

end

$v_0^{(0)} = 1$

for j=1 to n do

for k=0 to *j*-1 do

$v_k^{(j)} = (t_k - t_j)w_k$

end

$$v_j^{(j)} = \prod_{k=0}^{j-1}(t_j - t_k)$$

end

for j=0 to n do

$$v_j^{(j)} = v_k^{(j)}/(t - t_j)$$

end

This algorithm performs the same operations as (3.7), only in a different order. A remarkable advantage of formula (3.7) is that the quantities that have to be computed in $\mathcal{O}(m^2)$ and $\mathcal{O}(n^2)$ operations do not depend on the data $f(x_i, t_j)$.

Another advantage of formula (3.7) that it does not depend on the other order in which the nodes are arranged.

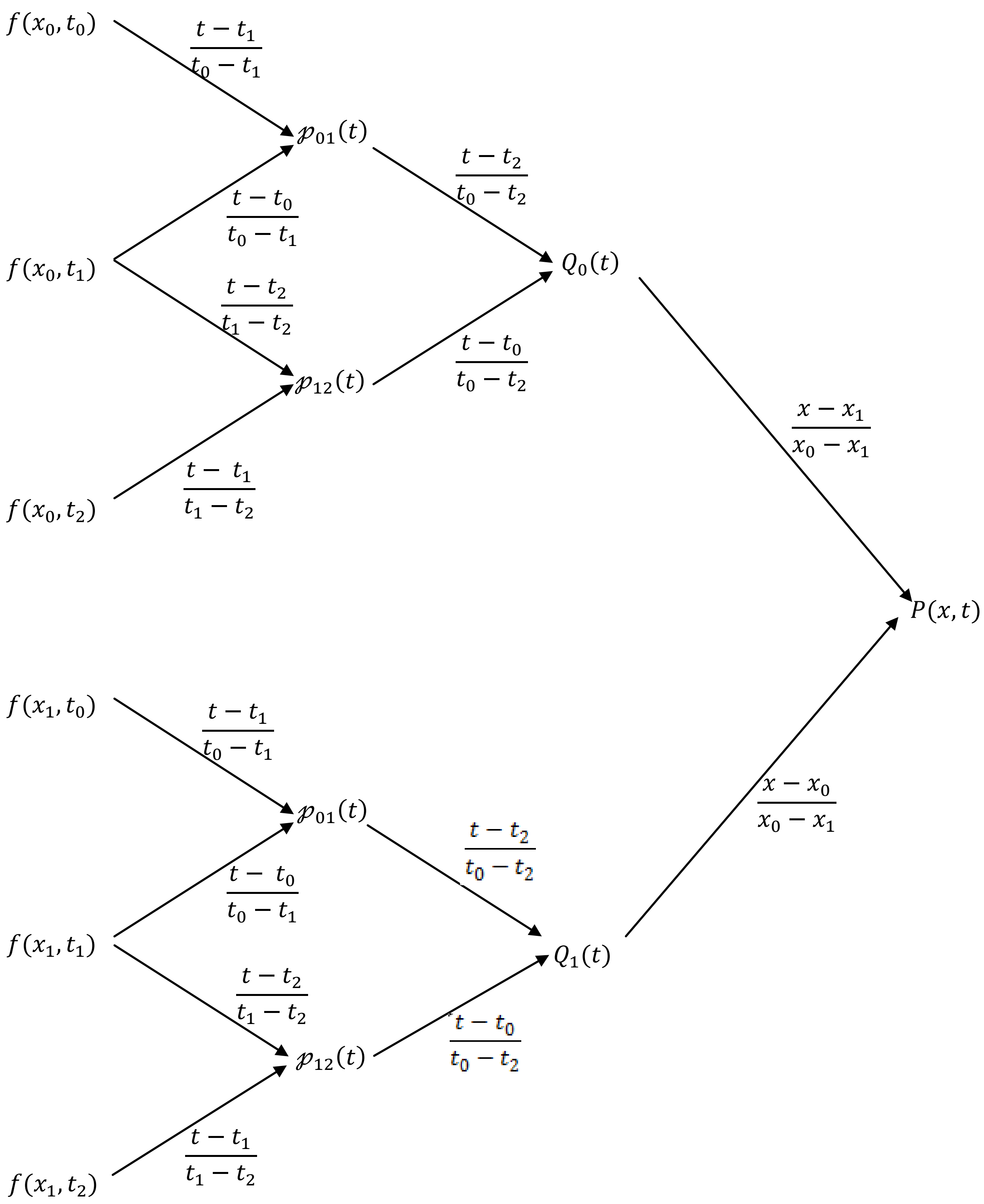
Now we suggest other modification that does not require an explicit representation of the polynomial, but we need only the values of the polynomial at specified points.

3.5. New Modification for Interpolation in Two Dimensions

In this section, another modification of interpolation in two dimensions. We will now illustrate a practical algorithm of this modification and we start by the following notification.

Notation: Let f be a function given at (x_0,t_0), (x_0,t_1), (x_0,t_2),…,(x_1,t_0), (x_1,t_1)… ,(x_m,t_n) and suppose that m_1, m_2, …., m_k are k distinct integers, with $0 \le m_i \le m$ for each i, and n_1, n_2, …, n_s are distinct integers, with $0 \le n_j \le n$ for each j. The two dimensions-interpolation polynomial that agrees with $f(x, t)$ at the (k, s) points (x_{m_1},t_{n_1}), (x_{m_1},t_{n_2}), (x_{m_1},t_{n_3}),…, (x_{m_2},t_{n_1}),…, (x_{m_2}, t_{n_2}), …, (x_{m_k}, t_{n_s}) is denoted by $Ƥ_{m_1,m_2,\ldots,m_k;n_1,n_2,\ldots n_s}(x, t)$.

The idea can be illustrated by using interpolation with one missing coordinates using the given data, which illustrated in the following diagram:


$f(x_0, t_0)$
$\frac{t-t_1}{t_0-t_1}$
$\wp_{01}(t)$
$\frac{t-t_2}{t_0-t_2}$
$\frac{t-t_0}{t_0-t_1}$
$f(x_0, t_1)$
$Q_0(t)$
$\frac{t-t_2}{t_1-t_2}$
$\frac{t-t_0}{t_0-t_2}$
$\wp_{12}(t)$
$\frac{x-x_1}{x_0-x_1}$
$\frac{t-t_1}{t_1-t_2}$
$f(x_0, t_2)$
$P(x,t)$
$f(x_1, t_0)$
$\frac{t-t_1}{t_0-t_1}$
$\frac{x-x_0}{x_0-x_1}$
$\wp_{01}(t)$
$\frac{t-t_2}{t_0-t_2}$
$\frac{t-t_0}{t_0-t_1}$
$f(x_1, t_1)$
$Q_1(t)$
$\frac{t-t_2}{t_1-t_2}$
$\frac{t-t_0}{t_0-t_2}$
$\wp_{12}(t)$
$\frac{t-t_1}{t_1-t_2}$
$f(x_1, t_2)$

Note, in this case, the function underlying the data might not be known so the explicit form of the error cannot be used.

Now, we introduce the algorithm, constructs the entries in suggested method by row. To evaluate the interpolation polynomial $\text{Ƥ}(\mathrm{x},\mathrm{t})$ on the $(m+1)(n+1)$ in distinct data (x_0,t_0), (x_0,t_1), (x_0,t_2),…, (x_1,t_0), (x_1,t_1), … (x_m,t_n) for the function f:

INPUT data x, t, x_0,t_0, x_1,t_1, … x_m,t_n ;values $f(x_0,t_0)$, $f(x_1,t_1)$, … $f(x_m,t_n)$ at the first column $Q_{0,0}, Q_{1,0}, \dots, Q_{m,0}$ of Q.

OUTPUT the table Q with $\text{Ƥ}(x) = Q_{m,n}$

Step 1: For i=1, 2,…,m ; j= 1, …,n

For k=1, 2,…, i ; s=1,…, j

Set $$Q_{ij} = \frac{(x-x_{i-k})(t-t_{j-s})Q_{i,k-1}-(x-x_i)(t-t_j)Q_{i-1,j-1}}{(x_i-x_{i-k})(t_j-t_{j-s})}$$

Step 2: Output (Q) ;

Stop

The algorithm can be modified to allow for the addition of new interpolating data. For example, the inequality

$$|Q_{i,i} - Q_{i-1,i-1}| < \in$$

can be used as a stopping criterion, if the inequality is true, then $Q_{i,i}$ is reasonable approximation to $f(x)$. if the inequality is false, a new interpolation point, x_{i+1}, is added.

Advantages of the suggested modification

1. Numerically stable: since has nice property such:
    - Directly uses the given data
    - Not need to represent the polynomial in the basis 1, t, t^2, ……
2. Fast implementation
3. Simple Structure: since has:
    - Parallel property
    - Strip off first and last indices
4. Can be update easy.

Figure (3.4) and (3.5) illustrate how can applied the suggested modification on the study region.

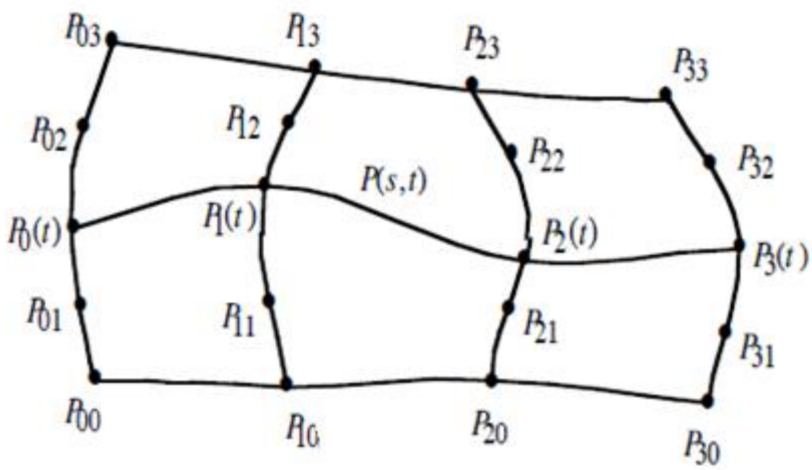


Figure 3.4: choose the data

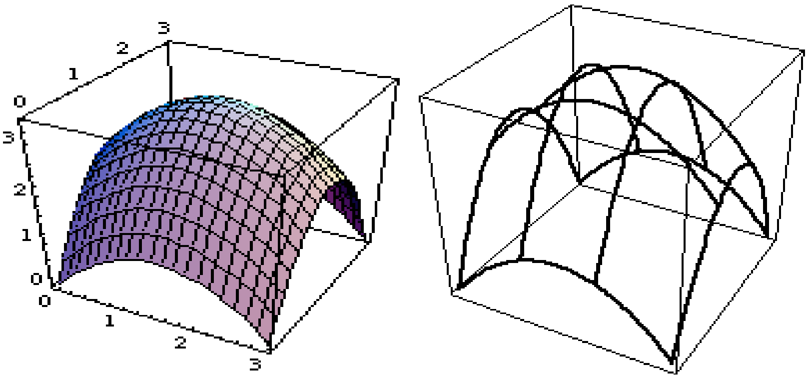


Figure 3.5: Surface Interpolation

Chapter Four

Applications

4.1. Introduction

In this chapter, we introduce an application for the suggested design of model equation and suggested methods which described dilation in previous chapters about the soil contamination by heavy metals for different zones in Baghdad city. Our aim to estimate the concentration of those heavy metals such Cd, Cu, Pb, Cr, Ni, Fe, Mn, and Co.

4.2. Sampling

The Capital of Iraq Baghdad city (33°14'-33°25'N, 44°31'-44°17'E), is characterized by arid to semi-arid climate with dry hot summers and cold winters; the mean annual rainfall is about 151.8 mm. For the purpose of collection of soil samples, the study area was divided into five main types of land use viz. residential, commercial, agricultural, main roads and industrial; and two main source areas, within each land use type viz. roadside and open areas. The sample areas are illustrated by geographic information system (GIS).

The samples are carefully collected from each source area in different land using a stainless steel spatula. They were air–dried in the laboratory, homogenized and sieved through a 2mm polyethylene sieve to remove large debris, stones and pebbles, after they were disaggregated with a porcelain pestle and mortar. Then these samples were stored in clean self–sealing plastic bags for further analysis.

Metal determinations were done by Atomic Absorption Spectrometry (AAS 6300, Shimadzu, Japan), X– ray fluorescence analysis (XRF) or Inductively Coupled Plasma-Mass Spectrometry (ICP-MS).

The samples are carefully chosen from each source area depending on the suggested technique in previous chapters.

Firstly, studying the pollution of soil by heavy metals was done in Al-Nahrawan zone; where the sampling was carried out in April 2015. Figure (4.1) gives an indication for the character of the zones, from which samples were taken.

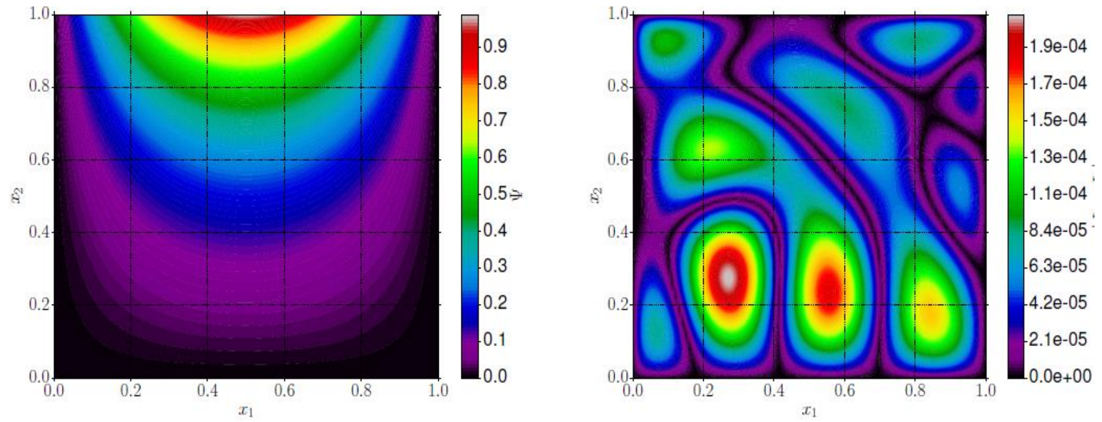
Samples were collected from 20 points, where ten soil samples have been collected with depth (0-10) cm, and other ten soil samples have been collected with depth (10-20) cm, using soil core, then all the samples were put in plastic bags to measure the concentration of heavy metals Cd, Cu, Pb, Cr, Ni, Fe, Mn, and Co.

The soil samples were taken on a dry day from various categories of gardens on road sides near factories, children's playgrounds, schools situated in the residential area, tanning leather factories and brick factories.

Figure 4.1: Location of samples in Al-Nahrawan by GIS

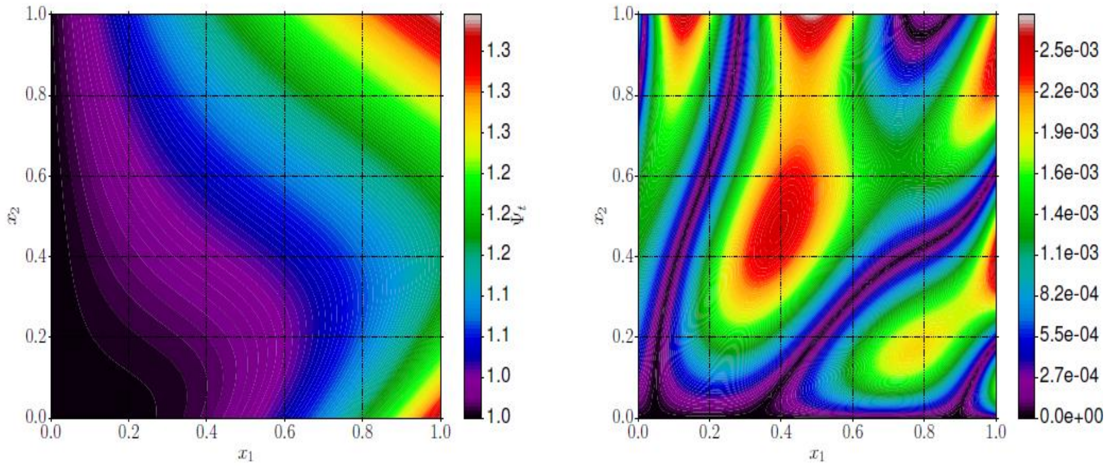
In the laboratory, samples were sieved in a 2-mm sieve to remove stones, glass and large plant roots and subsequently dried at room temperature for 3 days. The dried samples were then homogenized with a mortar and a pestle. The procedure described by [30] was applied to digest the samples with some modifications, and then all samples were taken to the laboratory to prepare them for analyzing by AAS equipment.

The interpolation method had been used to compute the concentrations of heavy metals in the study region. Suppose that a value of concentrations of any of heavy metals at a specific point in the study region without using the laboratory equipments but depending on the neighbor-hood points that the concentrations of heavy metals are known by using suggested interpolation method, then the results are given in Figure (4.2), which illustrates the distribution of heavy metal in the study region in Al-Nahrawan for (0-10cm) depth. Also, the concentrations of heavy metals in the study region for (10-20cm) depth depending on the solution of model equation which modified in chapter two and the results are illustrated in Figure (4.3).



(a) (b)

Figure 4.2:(a) Distribution of heavy metal in Al-Nahrawan with (0-10cm) depth (b)Comparison between standard and computed concentrations of heavy metals for zones in (a)



(a) (b)

Figure 4.3: (a) Distribution of heavy metal in Al-Nahrawan with (10-20cm) depth (b)Comparison between standard and computed concentrations of heavy metals for zones in (a).

Table 4.1: Standard Universal for concentration of heavy metals in soil [8]

Element	Standard Soils (mg/kg)	Critical Bound (mg/kg)	Pollution Soils (mg/kg)
Pb	50	150	600
Cd	1	5	20
Ni	50	100	500
Cr	100	250	800
Co	1	10	800
Zn	70	300	800
Cu	20	20	800
Fe	38000		

Table 4.2: The concentration of heavy metals in study regions with measure units PPm

Zone	Depth(cm)	Pb	Cr	Ni	Cd	Fe	Mn	Cu	Co
1	0-10	60	85	180	12	31500	200	36.5	13.5
2	0-10	55	80	173	12	31000	195	32	13
3	0-10	75	70	210	11	30500	195	27	13.5
4	0-10	75	75	203	13.5	31500	195	32	12
5	0-10	70	80	173	15	28000	185	27	12.5
6	0-10	70	90	180	14.5	36500	190	31.5	13
7	0-10	75	85	203	15	35000	180	29.5	13.5
8	0-10	70	80	188	12.5	29500	170	31.5	13
9	0-10	75	55	195	12	33000	190	31	12.5
10	0-10	80	75	203	13.5	32500	170	31.5	12.5
11	10-20	55	50	172.5	10	31000	195	32.5	13
12	10-20	60	75	165	11.5	29500	195	31	14
13	10-20	70	70	180	10	30000	185	26	12.5
14	10-20	60	72.5	180	12	30500	195	31	12
15	10-20	70	82.5	165	13	27000	185	26	11
16	10-20	65	87.5	150	12.5	35500	190	25.5	11.5
17	10-20	75	80	172.5	12.5	34500	180	27.5	12
18	10-20	60	80	150	12	26000	170	29	12.5
19	10-20	70	35	165	11.5	26500	190	22.5	11
20	10-20	75	70	175.5	12	30500	170	30.5	12
Mean	0-10	70.5	77.5	190.8	13.1	31900	187	30.95	12.9
	10-20	66	70.25	167.25	11.7	30100	185.5	28.15	12.15

The concentrations of heavy metals: Cd, Cu, Pb, Cr, Ni, Fe, Co and Mn for samples of depth (0-10cm) and (10-20cm) are given in Table (4.2). The mean concentration of Pb for depth (0-10cm) is 70.5 PPm and for depth (10-20cm) is 66 PPm which is higher than Pb concentration in the standard universal [8] that attain 50 PPm (see Table 4.1). Also, the concentration of Pb in depth (0-10cm) is higher than in depth (10-20cm).

The mean concentration of Cr for the depth (0-10cm) is 77.5 PPm but for the depth (10-20cm) is 70.25 PPm, the reason of increasing Cr concentration in soil is relating to the projectiles from chimneys of brick factories which contain high concentration of Cr when reaching soil after falling from air, and remnants of burning fuel, in addition to leather tanning factories where Cr is a basic element used in tanning leathers.

The mean of concentration of Ni in depth (0-10cm) is 190.8 PPm but in depth (10-20cm) is 167.2 PPm, that is higher than the standard universal were attain 50 PPm (see Table 4.1). The mean of concentration of Cd in depth (0-10cm) is 13.1 PPm and in depth (10-20cm) is 11.7 PPm.

The mean of concentration of Fe for the depth (0-10cm) is 31900 PPm but in depth (10-20cm) is 30100 PPm, i.e., the concentration of Fe in soil region is little bit more than the existing normal Geochemistry, also its concentration in depth (0-10cm) is higher than the concentration in depth (10-20cm) and the agricultural lands consist high concentration of Fe . The mean of concentration of Mn in depth (0-10cm) is 187 PPm and in depth (10-20cm) is 185.5 PPm, but the mean of concentration of Mn in clay is (200-1200 PPm) and in sand is (20-500 PPm). The Eh and PH have important spell for determining their removing [6].

The mean concentration of Cu in depth (0-10cm) is 30.95 PPm but in depth (10-20cm) is 28.15 PPm. But the mean of concentration of Co in depth (0-10cm) is 12.9PPm and in depth (10-20cm) is 12.15 PPm and both values are exceeding the standard universal which is 10 PPm (see Table 4.1).

Now, our methods will be applied to study the pollution of soil by heavy metals in Baghdad city generally. In this case, the data and information on soil contaminants have been selected from 12 stations located on different parts of the city of Baghdad for the purpose of collecting samples of soil have been distributed on a regular basis so we tried to cover most areas of the city, with a focus on the type of each area as residential, commercial, agricultural, main roads and industrial; as shown in Figure (4.4).

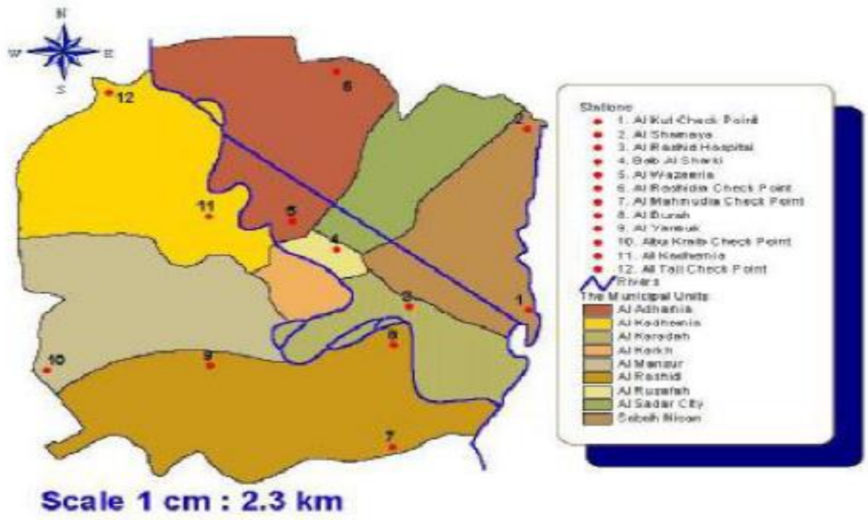


Figure 4.4: Map of Baghdad city showing study stations

Applying the equation (2.14) in chapter two, by substituting the initial concentrations (C_0) of heavy metals which were obtained from the selected sample laboratory by using XRF equipment or obtained from previous researches. The solution then is the concentrations of those heavy metals for any time t(d) and any depth x(cm).

The practical results illustrated in Figure (4.5), where t = 0, …, 5000 day and x = 0, 10, 20 cm, then the figure represent the concentrations of heavy metals in soil of Baghdad city for years from 2004 to 2016.

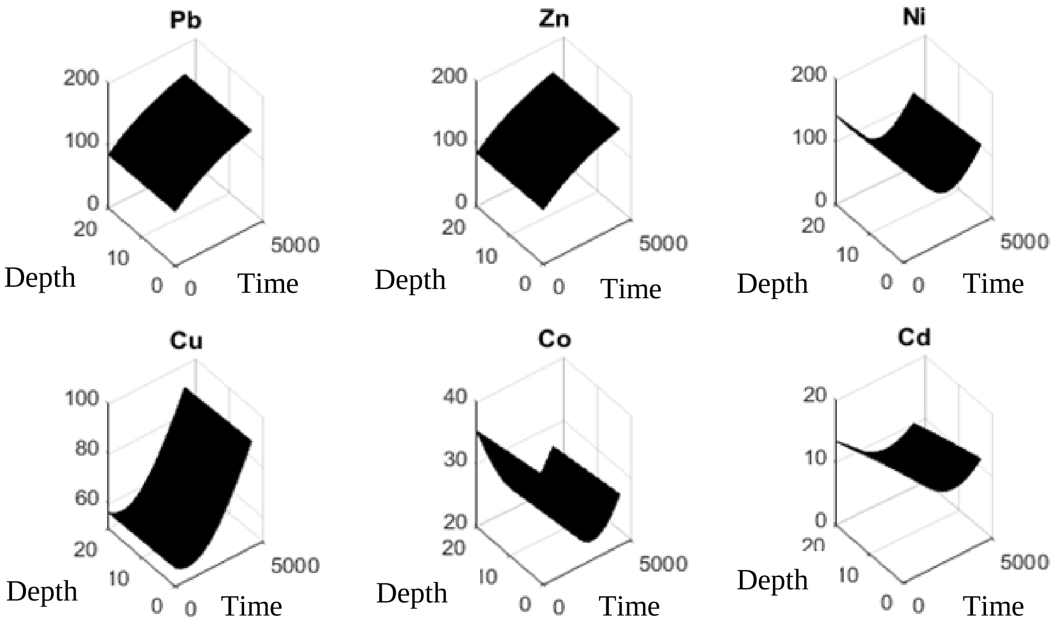


Figure 4.5: Concentrations of heavy metals for time in days and depth in centimeters in Baghdad city

Also, when applying the suggested methods to estimate the concentration of heavy metals in Baghdad soils but in this case, the data that used were for concentrations of previous years (1998 – 2014) from ministry of agriculture and the data for concentrations of 2015 are calculated in laboratory by using ICP-MS device and the results are illustrated as follows:

1. For spreading the Cu, which illustrated in Figure (4.6), represents kink waves which rise or descend from one asymptotic state to another and approaches a constant at infinity[41].

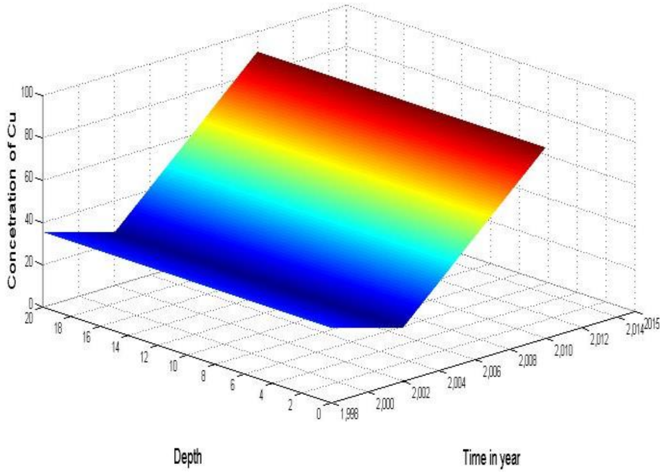


Figure 4.6: The wave solution for concentration of Cu in soil of Baghdad.

2. For spreading the Cd, which illustrated in Figure (4.7), represents kink waves.
3. For spreading the Zn, which illustrated in Figure (4.8), represents Peakons are peaked solitary wave solution. In this case, the wave solution is smooth except for a peak at a corner of its crest or for bottom. Peakons are the points at which spatial derivative changes sign so that peakons have a finite jump in first derivative of the solution C(x,t). This means that peakons have discontinuities in the *x*-derivative but both one-sided derivatives exist and differ only by a sign [33].

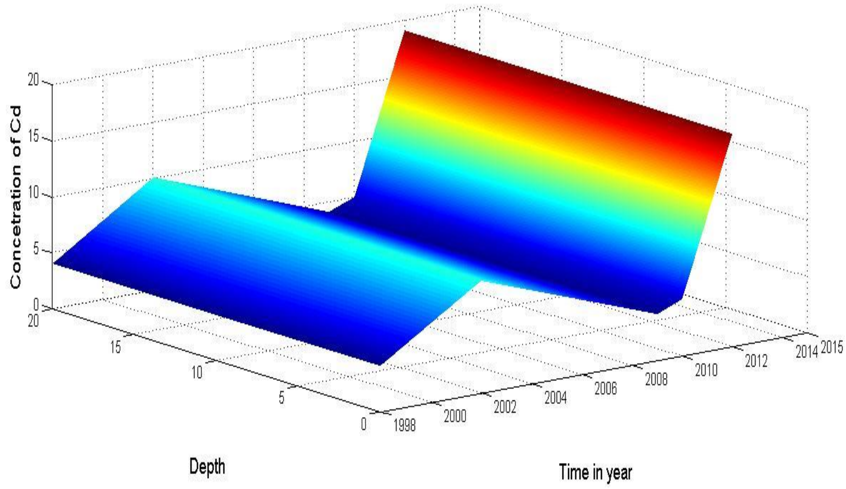


Figure 4.7: The wave solution for concentration of Cd in soil of Baghdad

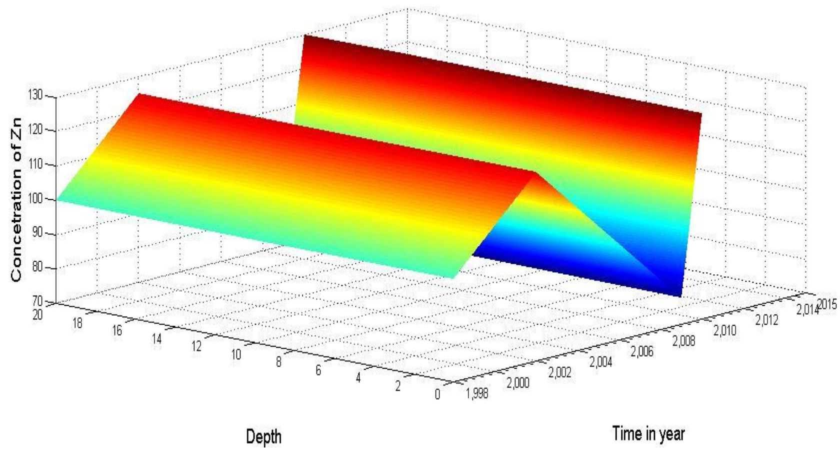


Figure 4.8: The wave solution for concentration of Zn in soil of Baghdad

4. For spreading the Pb, which illustrated in Figure (4.9), represents kink waves in the first part which defined on the interval [1998, 2011] and represents peakon waves in the second part defined on remainder interval.

5. For spreading the Co, which illustrated in Figure (4.10), represents kink waves.

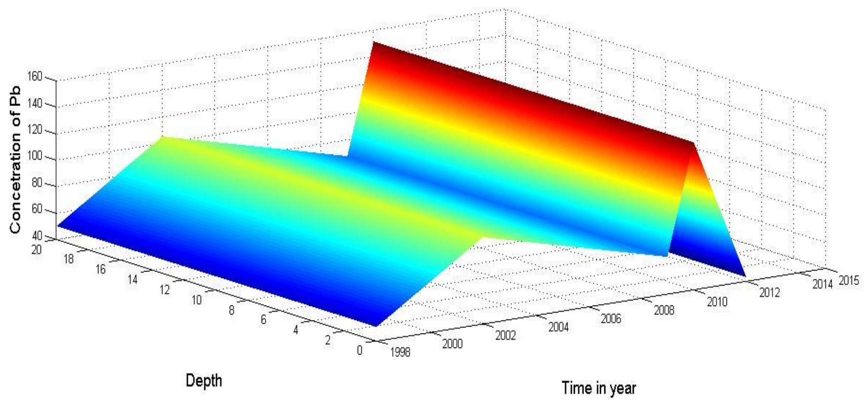


Figure 4.9: The wave solution for concentration of Pb in soil of Baghdad

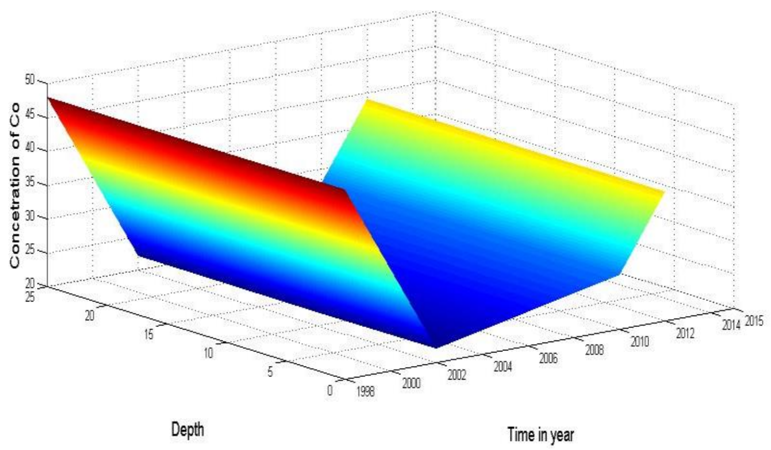


Figure 4.10: The wave solution for concentration of Co in soil of Baghdad

6. For spreading the Ni, which illustrated in Figure (4.11), represents kink waves.

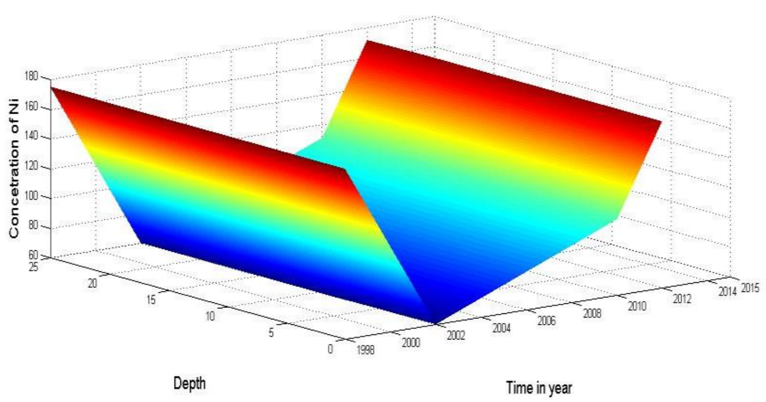


Figure 4.11: The wave solution for concentration of Ni in soil of Baghdad

Now, we use our suggested method to estimate the spread of the heavy metals in soil of different zones in Baghdad such as: residential, agricultural, commercial and industrial zones.

Note, the soil samples are taken of agricultural zones in Al-Jadiriya, Qanat Al-Jaeesh, and Al- Zafaraniyah.

The laboratory results for many zones in Baghdad are given in Table (4.3 – 4.6). The data for previous years (1998, 2002, 2009, 2010 and 2012) are given in Tables (4.7,4.9). Table (4.8) illustrates the composition and properties of soil which used in the present study.

Figures (4.12), (4.13) and (4.14) illustrate the estimation of the concentration of heavy metals of depth (20–40) cm in agricultural lands for Qanat Al jaeesh, Al-Jadriya and Al- Zafaraniyah respectively. Figures (4.15), (4.16) and (4.17) illustrate the estimation of the concentration of heavy metals of depth (20–40) cm for many zones in Baghdad of residential, commercial and industrial zones respectively.

Table 4.3: The laboratory results for some zones in Baghdad with depth of 0-20 cm

Zones	Zn	Pb	Ni	Cd
Kasra wa Atash	8.35	–	28.85	0.15
Naseb Alshaheed zone	52.5	15.65	14	0.7
Rashidiya zone	15	6.3	47.75	0.4
Mahmudiyah zone	130	95	38.8	3
Abu Ghraib zone	37.7	12	20.5	-
Zuwarah Park	20	10.4	43.5	0.8
Al-Obeidi zone	135	18.35	25.6	0.05
Shaab zone	30.5	17.5	67.5	0.3

Table 4.4: The laboratory results for both sides of agricultural zones in Qanat Al jaeesh

Samples	Depth	Cu	Zn	Pb	Ni	Cd
State 1	0 – 20 cm	10	24	18.8	62.5	0.2
	20 – 40 cm	4.25	9	11	12.5	0.1
State 2	0 – 20 cm	8.5	14.5	11.6	57.5	0.1
	20 – 40 cm	6	14	10	47.5	0.1
State 3	0 – 20 cm	10	16	4.8	27.3	0.1
	20 – 40 cm	9.2	16.1	3.45	25	0.1
State 4	0 – 20 cm	8.6	15	24.05	43.5	0.6
	20 – 40 cm	8.7	10	10	13.75	0.35
State 5	0 – 20 cm	0.2	0.8	2.25	36.5	0.25
	20 – 40 cm	5.5	11.2	8	36	0.45
Average	0 – 20 cm	7.46	14.06	12.3	45.46	0.25
	20 – 40 cm	6.73	12.06	8.49	26.95	0.22

Table 4.5: The laboratory results for agricultural zones in Al-Jadriya

Samples	Depth	Cu	Zn	Pb	Ni	Cd
State 1	0 – 20 cm	12.5	85	12	55	0.2
	20 – 40 cm	12	32.5	10	37.5	0.2
State 2	0 – 20 cm	8	9.5	10	20.5	0.05
	20 – 40 cm	7	7	8	13	0.05
State 3	0 – 20 cm	8.5	11	11	26	0.05
	20 – 40 cm	10.5	14.5	10	13	0.05
State 4	0 – 20 cm	17.5	15.5	17.5	27	0.05
	20 – 40 cm	12.35	10	10	54	0.6
State 5	0 – 20 cm	9.35	10	13	54	0.75
	20 – 40 cm	12.6	12.5	11.5	55	0.8
Average	0 – 20 cm	11.17	26.2	12.7	36.5	0.22
	20 – 40 cm	10.89	15.3	9.9	34.5	0.34

Table 4.6: The laboratory results for agricultural zones in Al- Zafaraniyah

Samples	Depth	Cu	Zn	Pb	Ni	Cd
State 1	0 – 20 cm	9.5	25	34.5	52	0.25
	20 – 40 cm	6	10	6.1	55	0.15
State 2	0 – 20 cm	11	18.8	9.7	66	0.7
	20 – 40 cm	12	25	8.8	80	0.2
State 3	0 – 20 cm	4.0	18	7.65	9.5	0.65
	20 – 40 cm	1.00	7	2.75	2.5	0.1
Average	0 – 20 cm	8.167	20.6	17.283	42.5	0.533
	20 – 40 cm	6.333	14	5.8833	45.833	0.15

Table 4.7: Average of the concentrations of heavy metals in Baghdad soil.

HM	١٩٩٨ Depth 10 cm	٢٠٠٢ Depth 10 cm	٢٠٠٩ Depth 15 cm	٢٠١٠ Depth 20 cm	٢٠١٢ Depth 10 cm
Pb	٥٠	١٠٥	67.52	١٥١	٤٣
Zn	١٠٠	١٢٥	77.9	١٣٠	-
Cd	٤	١٠	4.11	٥	١٩
Cu	٣٥	٢٥	-	٩٠	-
Ni	١٧٥	٦٠	-	١١١	١٧٢
Co	٤٨	٢٢	-	٢٨	٣٩

Table 4.8: Composition and properties soil used in the present study.

Property	Soil
Particle size distribution (ASTM D 422)	
Sand (%)	1.5
Silt (%)	63.5
Clay (%)	35
Cation Exchange Capacity (meq/100g)	12.5
Initial pH	8.3
Background concentration of cadmium (mg/kg)	nill
Background concentration of nickel (mg/kg)	3.52
Background concentration of lead (mg/kg)	15
Organic matter (%)	0.49
Organic carbon (%)	0.16
Electrical conductivity EC (µS/cm)	593
Surface area (m^2/g)	22.776
Bulk density (g/cm^3)	1.1317
Porosity (n)	0.493
Specific weight	2.69
Soil classification	Silty clay loam

Table 4.9: The concentrations of HMs in Baghdad soil for 2012 and 20 cm depth

No.	site	Co	Cd	Ni	Pb
1.	**Shaab**	**41**	**19**	**١٤٤**	**٣٩**
2.	**Talbiya**	**37**	**15**	**١٦٩**	**٣٣**
3.	**Jamela**	**44**	**18**	**١٨١**	**٤١**
4.	**Adhmiya**	**36**	**11**	**١٨٣**	**٣٢**
5.	**Al-Salam**	**38**	**18**	**١٨٣**	**٣٨**
6.	**Ataifia**	**41**	**13**	**١٥٦**	**٣٤**
7.	**Haifa**	**58**	**41**	**١٨٠**	**١٨٣**
8.	**Zayuna**	**32**	**17**	**١٥٦**	**٣٦**
9.	**Baghdad jadeda**	**48**	**33**	**١٣٨**	**٥٨**
10.	**Mashtal**	**36**	**15**	**١٣٠**	**٣٦**
11.	**Zufaraniya**	**34**	**13**	**٢٠٨**	**٣٩**
12.	**Dora**	**33**	**17**	**١٠٥**	**٣٧**
13.	**Karada**	**35**	**16**	**١٨٦**	**٣٩**
14.	**Jaderiya**	**37**	**14**	**١٩٥**	**٤٩**
15.	**Abu Dsheer**	**51**	**32**	**١٦٩**	**٧٥**
16.	**Saidiya**	**38**	**18**	**١٦٩**	**٣٨**
17.	**Qadisiya**	**44**	**29**	**١٥٦**	**٥٩**
18.	**Mansor**	**38**	**14**	**١٨١**	**٣٦**
19.	**Qazaliya**	**39**	**14**	**١٢٩**	**٣٣**
20.	**Al jameaa**	**38**	**17**	**٢٠٨**	**٣١**
21.	**Khadhraa**	**33**	**15**	**١٥٧**	**٢٩**
22.	**Nafaq shurta**	**31**	**16**	**١٤٤**	**٣٧**
23.	**Amriya**	**33**	**17**	**١٥٦**	**٢٩**
24.	**Jehad**	**40**	**19**	**٢٠٨**	**٣٧**
25.	**Furat**	**31**	**17**	**١٣٠**	**٣٣**

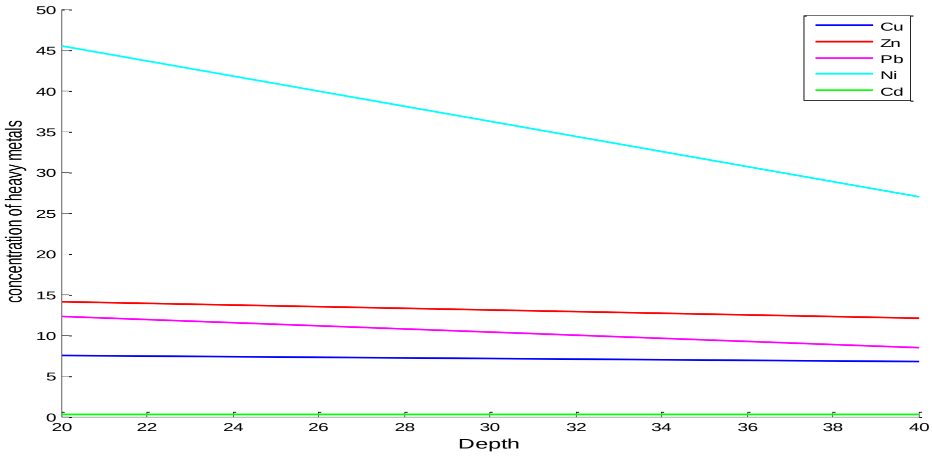


Figure 4.12: Concentration of heavy metals in agricultural lands for Qanat Al jaeesh

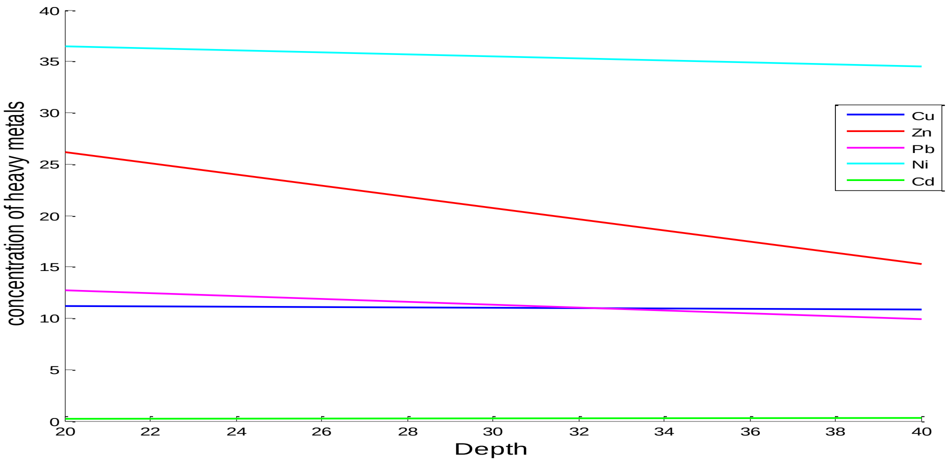


Figure 4.13: Concentration of heavy metals in agricultural lands for Al-Jadriya

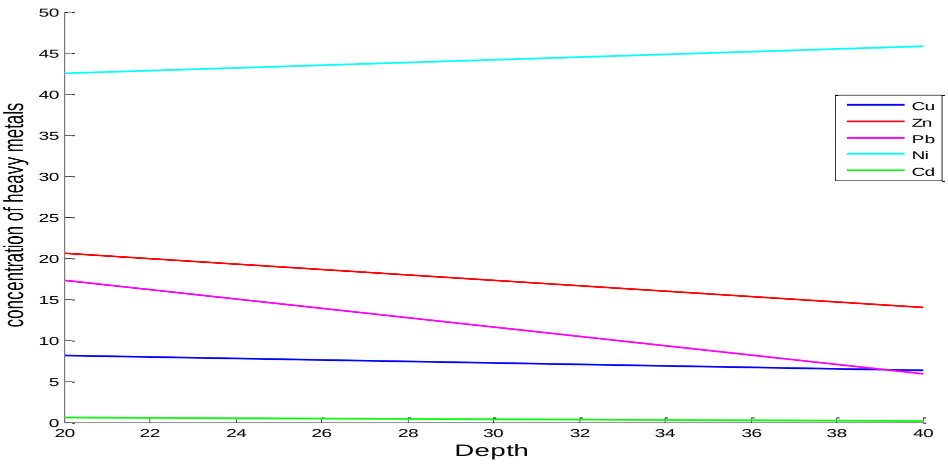


Figure 4.14: Concentration of heavy metals in agricultural lands for Al- Zafaraniyah

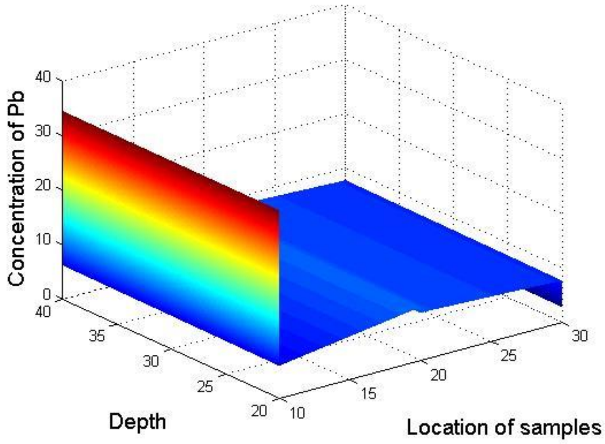


Figure 4.15: Concentration of heavy metals in residential lands for Baghdad

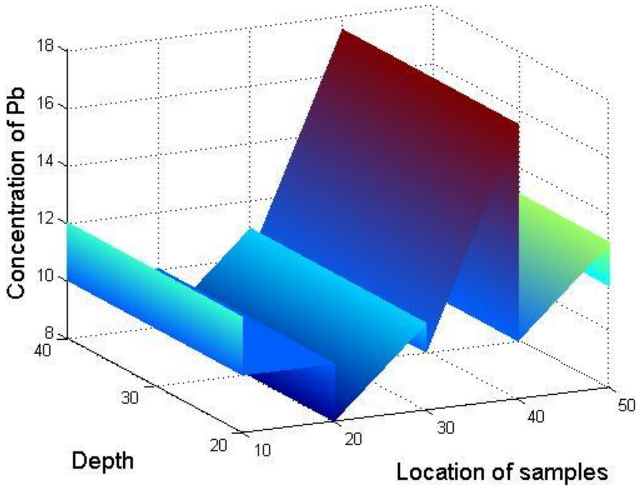


Figure 4.16: Concentration of heavy metals in commercial lands for Baghdad

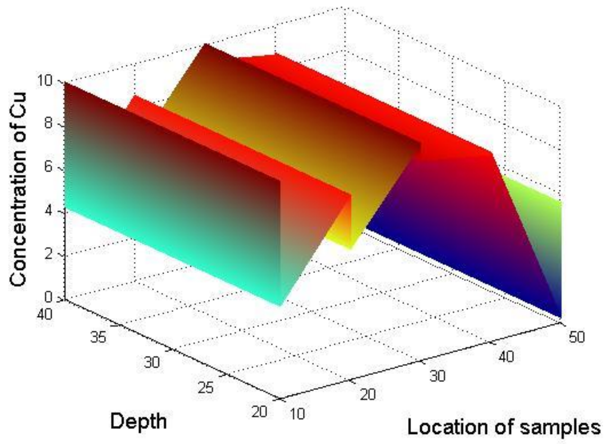


Figure 4.17: Concentration of heavy metals in industrial lands for Baghdad

4.3. Traditional Remediation of Contaminated Soil

Traditional treatments for metal contamination in soils are expensive and cost prohibitive when large areas of soil are contaminated. Treatments can be done *in situ* (on-site), or *ex situ* (removed and treated off-site). "Both are extremely expensive. Some treatments that are available include:

1. High temperature treatments (produce a vitrified, granular, non-leachable material).
2. Solidifying agents (produce cement-like material).
3. Washing process (leaches out contaminants).

The following management practices will not remove the heavy metal contaminants, but will help to immobilize them in the soil and reduce the potential for adverse effects from the metals, note that the kind of metal (cation or anion) must be considered:

1. Increasing the soil PH to 6.5 or higher.
2. Draining wet soils.
3. Applying phosphate.
4. Carefully selecting plants for use on metal-contaminated soils"[13].

In this thesis after details study and research, it is indicated that the locations which is moderate contaminated can be process naturally by the rain, others we suggest plants for treating metal contaminated in soils as follows.

4.4. Plants for Treating Metal Contaminated Soils

Plants have been used to stabilize or remove metals from soil. There are three mechanisms can be used for this purpose which are *phytoextraction*, *rhizofiltration*, and *phytostabilization* [21].

Rhizofiltration is the adsorption onto plant roots or absorption into plant roots of contaminants that are in solution surrounding the root zone (rhizosphere).

Phytostabilization is the use of perennial, non-harvested plants to stabilize or immobilize contaminants in the soil and groundwater.

Phytoextraction is the process of growing plants in metal contaminated soil. Plant roots then translocate the metals into aboveground portions of the plant. After plants have grown for some time, they are harvested and incinerated or composted to recycle the metals. Several crop growth cycles may be needed to decrease. Phytoextraction is done with plants called hyperaccumulators [13].

4.5. Discussion

The practical results shown as follows:

- The average of the concentrations of heavy metals in soil for different zones in Baghdad city increases with time, posing a great risk to the environment contamination.
- For the comparison among the concentrations of different regions: residential, industrial, commercial and agricultural regions, it is found that:

soils agricultural < soils residential < soils commercial < soils industrial that is, the agricultural regions are the lowest while the industrial regions are the highest for the concentrations of heavy metals.

- For comparing upon the depth of soil, we see the effect of depth on the concentration of heavy metals, that is, while the depth increase the concentrations of heavy metals are decrease, i.e., the concentration of heavy metals in depth (0–20) cm is larger than the concentration in depth (20–40) cm for the same soil.
- There are different causes for increasing the concentrations of heavy metals in soil such as: the big traffic jams resulting from the great number of cars lately which use gasoline that contains a lot of Fourth Lead Ethylene which cause big problem to the environment. This

creates dangers to human beings. In addition, the increase in the amount of litter and how to get rid of industry waste in sewerage and the decrease in the green region which participate in lessening the damage of waste on the environment.

As a result of increase in the population during the late years which results in converting the regions of vegetation to residential regions and the technological development which causes contamination because of the prolife ration of plants and workshops scattered everywhere. Add to all this, wars and their great contamination which are considered the most dangerous contaminates of the soil and environment.

All these types of contaminates cause high rate of concentration of waste which exceeds the normal amount in soil, the increase of these metals has different types of danger on human health.

The plants absorb these dangerous materials which in its turn go to human beings through food consumption which they acquire because of eating these plants that have the dangerous metals.

Chapter Five

Conclusions &Future Works

5.1. Conclusions

This thesis demonstrates the development of a model equation that describes the spread of heavy metals in soil vertically for any depth and solved this model equation by using ADM. Also, suggesting a numerical model which describes the spread of heavy metals in soil horizontally. These models can be used to estimates the concentration of heavy metals in soil for any depth and time. Thus the results provided many of the features, such as:

1- The model developed can be considered to be a good representation of that estimate the concentrations of heavy metals in the soil.

2- The suggested modification for interpolation in two dimensions is more efficient, easy implemented and rapid.

3- Figure (5.1) represent the concentrations of heavy metals in soil for different regions of Baghdad city.

4- The concentration of heavy metals can be estimated by interpolation technique. The sixty four soil samples from the different sections of Baghdad city were estimated by this technique. The prediction errors of this technique are less than 0.0002 compared with those of AAS or XRF. This technique is fast, convenient, sensitive, and can eliminate the interference among various species. The determination of sample by AAS is finished in seven days; while the determination of sample by the proposed technique is finished in the same minute. The study present from the chemical analysis of soil. This soil's region containing high concentrations of heavy metals because of projectiles falling of brick factories on that zone where loam metals and organic materials in that soil adsorbs these metals and increase their concentrations.

5- The concentration of the heavy metals in depth (0-20cm) is higher than the concentration in depth (20-40cm), since it has a direct contact with the projectiles of brick factories, increasing the organic materials in it and existence of loam metals which adsorb these studied heavy metals.

6- The practical results showed that the predictive models established in this thesis are much more efficient compared to those established by the traditional methods.

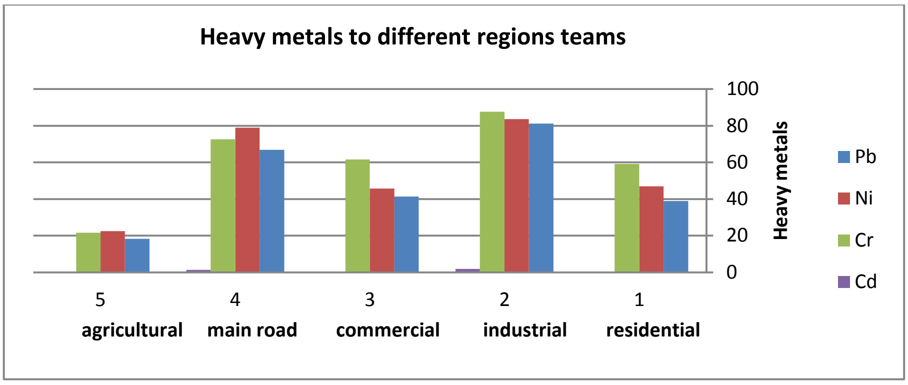


Figure 5.1: Comparison for concentration of heavy metals in Baghdad soil

5.2. Future Works

The studies for future investigations are suggested below:

1- Develop a model equation that describes other types of soil.

2- The researchers can use the modified mathematical model with the temperature as a variable, which may extend the equation to three dimensions.

3- Study the effect of Atmospheric pressure and winds, as variables when counting the concentrations on the surface of soil.

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المُستخلص

تضمنت هذه الرسالة اهدافاً عديدة:

الهدف الأول هو تطوير نموذج معادلة تصف انتشار المعادن الثقيلة في التربة عمودياً لأي عمق وأي زمن، ثم حل نموذج المعادلة ذاته باستخدام طريقة ADM.

الهدف الثاني الذي تضمنته هذه الرسالة هو اقتراح نموذج عددي يصف انتشار المعادن الثقيلة في التربة أُفقياً.

ويمكن استخدام النموذج المقترح لتقدير تراكيز العناصر الثقيلة في التربة لأي زمن. لذا، يمكن اتخاذ هذا النموذج كطريقة لتحديد مستويات التلوث من قبل بعض العناصر الثقيلة مثل النحاس، الحديد، الزنك، الكادميوم، الكوبلت والنيكل مع امكانية تحديد مصدر التلوث وتأثيرهم.

الهدف الثالث من الرسالة هو تطوير المفهوم النظري للإندراج ذو البعدين ثم توضيح دقة، كفاءة، سرعة، وسهولة تنفيذ الطريقة المقترحة.

الهدف الرابع هو مقارنة أداء النموذج المقترح مع الطرق التقليدية، وذلك من خلال حساب تراكيز المعادن الثقيلة في التربة ثم تطبيقها على تربة بغداد بعد تصنيفها الى

مناطق مختلفة مثلاً: مناطق سكنية، صناعية، تجارية، زراعية، وطرق رئيسة. ثم مقارنة نتائج النماذج المقترحة مع النتائج التي حصلنا عليها من خلال الفحص المختبري باستخدام جهاز الإمتصاص الذري، وجهاز الفحص بالأشعة السينية، وجهاز بلازما الطيف الكتلي، لتحديد نسبة الدقة. أظهرت نتائج العمل أنّ النموذج المُقترح يمكن تطبيقه بنجاح بالنظر الى السرعة والدقة في تقديره لتراكيز العناصر الثقيلة في التربة.

أخيراً، اقترحنا بعض طرق علاج التلوث في التربة باستخدام بعض النباتات العشبية.

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
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تخمين نسبة التلوث في التربة باستخدام طرق رياضية مطورة

رسالة
مقدمة إلى كلية التربية للعلوم الصرفة / ابن الهيثم – جامعة بغداد
وهي جزء من متطلبات نيل شهادة ماجستير علوم
في الرياضيات

من قبل
ايمان عدي عبدالحميد

بإشراف
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